Prove each of the following statements. (Please make sure to write your answers in complete sentences.)

1. Prove the following formula by induction:
   \[ 1 + 3 + 5 + \cdots + (2n - 1) = n^2 \]

2. Let \( V \) be a linear space. In particular we know that for all \( a \in \mathbb{R} \) and for \( x, y \in V \), \( a(x + y) = ax + ay \). Prove by induction that, for all \( a \in \mathbb{R} \) and for all \( x_1, \ldots, x_n \in V \),
   \[ a(x_1 + \cdots + x_n) = ax_1 + \cdots + ax_n. \]

3. Let \( P(n) \) be the statement
   \[ 1 + 2 + \cdots + n = \frac{1}{8}(2n + 1)^2. \]
   (a) Prove that if \( P(k) \) is true for an integer \( k \), then \( P(k + 1) \) is also true.
   (b) Criticize the statement “By induction, it follows that \( P(n) \) is true for all \( n \)”