Read the following, which can all be found either in the textbook or on the course website.

- Chapters 1 & 2 of *Visual Group Theory* (VGT).
- VGT Exercises 1.1–1.4, 1.8–1.12, 2.13–2.17.
- The article *Group Think* by Steven Strogatz, which appeared in the NY Times in 2010.

Write up solutions to the following exercises.

1. Given a regular \( n \)-gon, let \( r \) be a rotation of it by \( 2\pi/n \) radians. This time, assume that we are not allowed to flip over the \( n \)-gon. These \( n \) actions form a group denoted \( C_n = \langle r \rangle = \{e, r, r^2, \ldots, r^{n-1}\} \).

   (a) Draw a Cayley diagram for \( C_n \) for \( n = 4, n = 5, \) and \( n = 6 \).
   (b) For \( n = 4, 5, 6 \), find all minimal generating sets of \( C_n \).
   (c) Make a conjecture of what integers \( k \) does \( C_n = \langle r^k \rangle \) for a general fixed integer \( n \).

2. As we saw in lecture, the six symmetries of an equilateral triangle \( \triangle \) form a group denoted \( D_3 = \{e, r, r^2, f, rf, r^2f\} \), where \( r \) is a 120° clockwise rotation and \( f \) is a flip about a vertical axis (which fixes the top corner). Since \( r \) and \( f \) suffice to generate all six of these symmetries, we write \( D_3 = \langle r, f \rangle \).

   (a) Let \( g \) be the reflection of the triangle that fixes the lower-left corner. Which of the six actions in \( D_3 \) is \( g \) equal to? Which action is \( fg \)?
   (b) Write all 6 actions of \( D_3 \) using only \( f \) and \( g \). Draw a Cayley diagram using \( f \) and \( g \) as generators.
   (c) To generate \( D_3 \), we need at least 2 actions. It is not difficult to show that if we have 3 generators, then one of them is unnecessary. Find all minimal generating sets of \( D_3 = \{e, r, r^2, f, rf, r^2f\} \); note that all of them should have exactly two actions. Do not use \( g \) in this list.

3. The eight symmetries of a square \( \square \) form a group denoted \( D_4 \). Let \( r \) be a 90° clockwise rotation and \( f \) a horizontal flip (that is, about a vertical axis). It is not difficult to show that \( D_4 = \langle r, f \rangle \).

   (a) Write all 8 actions of \( D_4 \) using \( r \) and \( f \) and draw a Cayley diagram using these two actions as generators.
   (b) Let \( g \) be the reflection of the square that fixes the lower-left and upper-right corner. Which of the eight actions in \( D_4 \) is \( g \) equal to? Which action is \( fg \)?
   (c) Draw a Cayley diagram of \( D_4 \) using \( f \) and \( g \) as generators.
   (d) Find all minimal generating sets of \( D_4 \). [Hint: There are 12.]
4. Pick any integer and consider this set of actions: adding any integer to the one you choose. This is an infinite set of actions; we might name them like “add 1” and “add $-4210$,” etc. This is a group. Find all minimal generating sets. Sketch a Cayley graph for this group using one of these minimal generating sets.