1. Let $E$ be an elliptic curve given in short Weierstrass form by
\[ y^2 = f(x) \]
where $f(x)$ is the polynomial $x^3 + ax + b$.

   a) Let $P_i = (x_i, y_i)$, $i = 0, 1$ be two points in $E(\mathbb{Q})$. Assume that $x_1 \neq x_2$ nd $y_1 \neq y_2$. Write out a formula for $P_1 + P_2$.
   
   b) Characterize the two torsion subgroup of $E(\mathbb{C})$ in terms of $f(x)$.
   
   c) Let $P_1, P_2, P_3$ be distinct points on $E(\mathbb{Q})$ and suppose that they lie on a line. Prove that $P_1 + P_2 + P_3 = 0$.

2. Browse through chapter of William Stein’s book, which is freely available here: http://wstein.org/ent/. Figure out how to
   
   – create examples of elliptic curves,
   
   – compute their ranks, and
   
   – compute their torsion subgroups.

Compute lots of examples, and answer the following questions:

   a) What was the largest rank that you found?
   b) What torsion subgroups did you find?

Your write up of this problem should include the answers to a and b and the Sage code used to find your answers.