MATH 490, HW 5 - MORE ABOUT LATTICES

**Due:** Wednesday, February 2

1. Using that
   \[\wp(z) = \frac{1}{z^2} + \sum_{n=1}^{\infty} (2k+1)G_{2k+2}z^{2k}\]
   show that
   \[\wp'(x)^2 = 4\wp(z)^3 - 60G_4\wp(z) - 140G_6.\]

2. Let \(\Lambda = \{a + bi : a, b \in \mathbb{Z}\}\). Show that the elliptic curve
   \[y^2 = 4x^3 - 60G_4\wp(z) - 140G_6\]
   is isomorphic to the elliptic curve \(E: y^2 = x^3 - x\). Hints:
   a) Use \(i\Lambda = \Lambda\) to show that \(G_6 = 0\).
   b) Show that \(G_4 \neq 0\). This is harder and will probably require a combination of
      computation in Sage, understanding the argument in the proof of [Sil09, Theorem
      3.1 (a) of Chapter VI], which is available here
      \[http://books.google.com/books?id=Z90CA_EUCkC&lpg=PR1&dq=elliptic%20curves%20silverman&pg=PA165#v=onepage&q&f=false\]
      and the fact that
      \[\sum_{1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.\]

**References**