(1) Let $C$ be a category and let $h: C \to \text{Fun}(C^{\text{op}}, \text{Sets})$ be the Yoneda embedding. Show that for any arrows $X \to Y$ and $Z \to Y$ in $C$, there is a natural isomorphism

$$h_{X \times_Y Z} \to h_X \times_{h_Y} h_Z$$

of functors, where $h_X \times_{h_Y} h_Z$ is the functor

$$h_X \times_{h_Y} h_Z: W \mapsto h_X(W) \times_{h_Y(W)} h_Z(W).$$

(2) Let $G$ be an object of a category $C$. Show that the functor of points $h_G: C^{\text{op}} \to \text{Sets}$ factors through the forgetful functor from groups

$$\begin{array}{ccc}
\text{Groups} & \to & \text{Sets} \\
\downarrow & & \downarrow \\
h_G: C^{\text{op}} & \to & \text{Sets}
\end{array}$$

if and only if $G$ is a group object of $C$.

(3) Let $\star$ be one of Spec $k$ (with $k$ a field) or Spec $\mathbb{Z}$. Let

$$G = \coprod_{g \in \mathbb{Z}/2\mathbb{Z}} \star$$

be the group object corresponding to $\mathbb{Z}/2\mathbb{Z}$. Work out explicitly the group structure. In other words, work out the maps in terms of rings, and show that $G$ represents the functor

$$\text{Sch}^{\text{op}} \to \text{Groups}, W \mapsto \mathbb{Z}/2\mathbb{Z}$$

(4) (a) Let $n \geq 1$ be an integer and let

$$\text{GL}_n: (\text{Sch})^{\text{op}} \to \text{Sets}$$

be the functor sending a scheme $Y$ to the set $\text{GL}_n(\Gamma(Y, \mathcal{O}_Y))$. Prove that $\text{GL}_n$ is a representable functor.

(b) Let $X$ represent the functor $\text{GL}_n$. Prove that the group structures on the sets $\text{GL}_m(\Gamma(Y, \mathcal{O}_Y))$ induce the structure of a group scheme on $X$. (I.e. use the previous exercise, noting that there is one detail to check.)

(5) Give an example of a category $C$, a subcategory $C'$, and a diagram $D: I \to C'$ such that the limit (or colimit, your choice) in $C'$ is not the limit in $C$.

(6) Use the functor of points to define a map $\mathbb{A}^1 \to \mathbb{A}^1$ given by the formula $z \mapsto z^2$. Compare this with how one would define such a map with locally ringed spaces.

(7) Monomorphisms.

(a) Show that a morphism $X \to Y$ is a monomorphism if and only if for every $T \in C$, the map of sets $X(T) \to Y(T)$ is injective.

(b) Show that a map of schemes which is injective topologically may not be a monomorphism.
(c) Show that a map of schemes which is surjective topologically may not be an epimorphism.