

STACKS HW5 - COMMA CATEGORY AND ADJUNCTION

- (1) Let C be a site, and let X be an object in C . Recall that the comma category C/X inherits the structure of a site. Assume that C is subcanonical (which means that for every $X \in C$, h_X is a sheaf).
- (a) Show that there is an equivalence of categories between $Sh(C/X)$ and $Sh(C)/h_X$.
 - (b) Show that $j^*: Sh(C) \rightarrow Sh(C)/h_X$, given by $F \mapsto (F \times h_X \xrightarrow{p_2} h_X)$ commutes with finite limits and has a right adjoint j_* . (Describe j_* explicitly.)