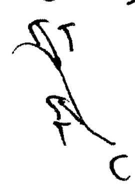
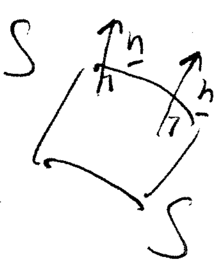
Curves	Surfaces
$\int_{[a,b]} f\,dx = \int_a^b f(x)\,dx$, $[a,b]$ = "flat curve in $\mathbb{R}$"	$\iint_D f(x,y)\,dA$, D = "flat" region in $\mathbb{R}^2$
Integral of f over a curve C: $\int_C f\,ds := \int_a^b f(\underline{r}(t))\,\lvert\underline{r}'(t)\rvert\,dt$	Integral of f over a surface S $\iint_S f\,dS = \iint_D f(\underline{r}(u,v))\,\lvert\underline{r}_u \times \underline{r}_v\rvert\,dA$
For a curve in the plane $\mathbb{R}^2$ which is the graph of a function $y=g(x)$, $a \le x \le b$ $\int_C f\,ds = \int_a^b f(t,g(t))\sqrt{1+[g'(t)]^2}\,dt$	For a surface that is the graph of a function $z=g(x,y)$ over a region D in the plane $\mathbb{R}^2$: $\iint_S f\,dS = \iint_D f(x,y,g(x,y))\sqrt{g_x^2+g_y^2+1}\,dA$
$L(C) = \int_C ds$ is the length of C (Same 2 formulas as above without f)	$A(S) = \iint_S dS$ is the area of S (Same two formulas as above without f
Integral of vector field $\underline{F}$ over a curve C: $\int_C \underline{F}\cdot d\underline{r} := \int_C (\underline{F}\cdot\underline{T})\,ds$, where T is a unit tangent vector to C.  If $\underline{F} = P\underline{i} + Q\underline{j} + R\underline{k}$ $\int_C \underline{F}\cdot d\underline{r} = \int_C P\,dx + Q\,dy + R\,dz =$ $= \int_a^b (Px' + Qy' + Rz')\,dt$	Integral of vector field $\underline{F}$ over a surface S: $\iint_S \underline{F}\cdot d\underline{S} := \iint_S (\underline{F}\cdot\underline{n})\,dS$, where $\underline{n}$ is a unit normal vector to S  If $F = P\underline{i} + Q\underline{j} + R\underline{k}$ $\iint_S \underline{F}\cdot d\underline{S} = \begin{cases} \iint_D F\cdot(\underline{r}_u \times \underline{r}_v)\,dA & \text{in parameterized case} \\ \iint_D [-Pg_x - Qg_y + R]\,dA & \text{for } S \text{ given by } z = g(x,y) \end{cases}$

Curves	Surfaces
"Flat curve" = interval $[a,b]$ $\int_{[a,b]} f\,dx$ is just $\int_a^b f(x)\,dx$	"flat surface" = region D in $\mathbb{R}^2$ $\iint_D f(x,y)\,dA$
Integral of a function f over a curve C, parameterized as $\underline{r}(t)$, $a \le t \le b$: $\int_C f\,ds = \int_a^b f(\underline{r}(t))\,\lvert \underline{r}'(t)\rvert\,dt$ where $\underline{r}(t) = (x(t), y(t), z(t))$ and $\lvert r'(t)\rvert = \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}$	Integral of a function $f(x,y,z)$ over a surface S, parameterized as $\underline{r}(u,v)$, with (u,v) in a region D in $\mathbb{R}^2$: $\iint_S f\,dS = \iint_D f(\underline{r}(u,v))\,\lvert \underline{r}_u \times \underline{r}_v\rvert\,dA$ where $\underline{r}(u,v) = (x(u,v), y(u,v), z(u,v))$ and $\lvert \underline{r}_u \times \underline{r}_v\rvert = \sqrt{(y_u z_v - z_u y_v)^2 + (x_u z_v - z_u x_v)^2 + (x_u y_v - y_u x_v)^2}$
For a curve in the plane $\underline{r}(t) = (x(t), y(t))$, $\underline{r}'(t) = \sqrt{[x'(t)]^2 + [y'(t)]^2}$. If $\underline{r}(t)$ is the graph of a function $y = g(x)$, then $\underline{r}(t) = (t, g(t))$ or $\underline{r}(x) = (x, g(x))$ and $\int_C f\,ds = \int_a^b f(t, g(t))\sqrt{1 + [g'(t)]^2}\,dt$	For a surface that is a graph of a function $z = g(x,y)$, then $\underline{r}(u,v) = (u, v, g(u,v))$ or $\underline{r}(x,y) = (x, y, g(x,y))$ and $\iint_S f\,dS = \iint_D f(x, y, g(x,y))\sqrt{g_x^2 + g_y^2 + 1}\,dA$
For $f \equiv 1$: $\text{length}(C) = L(C) = \int_C ds$	For $f \equiv 1$: $\text{Area}(S) = A(S) = \iint_S dS$
~~Line~~ Integral of a vector field $\underline{F}$ over a curve C: $\int_C \underline{F} \cdot d\underline{r} = \int_C \left(\underline{F} \cdot \frac{\underline{r}'}{\lvert \underline{r}'\rvert}\right) ds$ If $\underline{F} = P(x,y,z)\,\underline{i} + Q(x,y,z)\,\underline{j} + R(x,y,z)\,\underline{k}$ Then $\int_C \underline{F} \cdot d\underline{r} = \int_C P\,dx + Q\,dy + R\,dz =$ $= \int_a^b [P(x(t), y(t), z(t))\,x'(t) + Q(x(t), y(t), z(t))\,y'(t) + R(x(t), y(t), z(t))\,z'(t)]\,dt$ $= \int_a^b (P x' + Q y' + R z')\,dt$	Surface integral of a vector field $\underline{F}$ over a surface S: $\iint_S \underline{F} \cdot d\underline{S} = \iint_S (\underline{F} \cdot \underline{n})\,dS$ where $\underline{n}$ is a unit normal to the surface. If $\underline{F} = P(x,y,z)\,\underline{i} + Q(x,y,z)\,\underline{j} + R(x,y,z)\,\underline{k}$ then $\iint_S \underline{F} \cdot d\underline{S} = \begin{cases} \iint_D \underline{F} \cdot (\underline{r}_u \times \underline{r}_v)\,dA & \text{(for } S \text{ parameterized)} \\ \iint_D [-P g_x - Q g_y + R]\,dA & \text{(for } S \text{ given by } z = g(x,y)) \end{cases}$