

## COMPUTER GRAPHING OF A SYSTEM

The following description assumes one has installed the programs on the diskette attached to the book on a Windows system, and gives a detailed but direct example on how to graph the phase portrait of the following system:

$$\mathbf{x}' = \begin{pmatrix} -1/2 & 1 \\ -1 & -1/2 \end{pmatrix} \mathbf{x} \quad (\text{Example 1 in page 385}).$$

Further information is provided in “A Guide to the ODE Architect Tool”, a PDF file that comes with the software.

- (1) Run the ODE Architect Tool.
- (2) The upper left white subwindow is the editor in which the equation is to be typed. Enter the following two lines:

$$\begin{aligned} x' &= -x/2 + y \quad // \\ y' &= -x - y/2 \quad // \end{aligned}$$

Note how we did not use any vector notation, and used  $x, y$  for the variables, and not  $x_1, x_2$ . Nor did we need to write down explicitly the dependent variable  $t$ . Finally, operations such as division and multiplication need to be given explicitly (e.g.  $*$  is multiplication).

- (3) Click on Equations, and choose in that menu: Enter. Two black subwindows for graphing will appear on the right. We will work with the upper right one.
- (4) Each black subwindow appears with little “knobs” on its lower left. Choose the x-y knob.
- (5) There are four buttons arranged vertically to the right of the graph subwindow. Choose the top one and a menu appears, from which one chooses Scales...
- (6) For X-Scale, uncheck Auto Scale, then enter  $-3$  and  $3$  as Minimum and Maximum, respectively. Press Y-Scale, do the same, and then press OK. You will notice the change in the ranges of each axis.
- (7) Choose the button as in (5), and choose Markers... In the appearing window, click Add. Then check Horizontal and Vertical in the Lines portion, and click OK.
- (8) Choose the button as in (5), and choose Direction Fields. A plot of these will appear.
- (9) On left of the graph, choose the third button from the top. Then click anywhere on the graph. This will cause a trajectory to be plotted, passing through the point that was clicked. You can repeat this at other points to get more trajectories.

This is all. There are further options: one can print the graph, the lower black graphing box can be used to see other features (e.g. the y-t graph), one can find the equilibrium points. To demonstrate the latter, note that if the system

$$\begin{aligned} x' &= 4 - 2 * y \quad // \\ y' &= 12 - 3 * x^2 \quad // \end{aligned}$$

is entered in part (2) (see Example 2 on page 477), one can click on the lowest of the four buttons to the right of the graph subwindow. After that, clicking on the graph near where one expects to find an equilibrium point, will cause the program to mark such a point. Note by the way that for this system, drawing the trajectories as in (9) may produce program errors, but the graph will still be drawn.