2.4.4

Assume that $x_{ij}$ is the number of units sent from Warehouse $i$ to Outlet $j$. Then the constraints for each warehouse and outlet are:

\[
\begin{align*}
x_{11} + x_{12} + x_{13} + x_{14} & \leq 100 \\
x_{21} + x_{22} + x_{23} + x_{24} & \leq 150 \\
x_{31} + x_{32} + x_{33} + x_{34} & \leq 300 \\
x_{11} + x_{21} + x_{31} & \geq 120 \\
x_{12} + x_{22} + x_{32} & \geq 120 \\
x_{13} + x_{23} + x_{33} & \geq 120 \\
x_{14} + x_{24} + x_{34} & \geq 120
\end{align*}
\]

Moreover, outlet 2 cannot receive more units from warehouse 1 than from warehouse 2, and outlet 4 must receive at least half of its supply from warehouse 3. This gives us two additional constraints:

\[
\begin{align*}
x_{12} & \leq x_{22} \\
x_{34} & \geq (x_{14} + x_{24} + x_{34})/2
\end{align*}
\]

To compute the objective function, note that the storage charge is $6 per unit at Warehouse 1 and 2, and$12 per unit in Warehouse 3. Therefore the storage cost is $6(100 - x_{11} - x_{12} - x_{13} - x_{14}) + 6(150 - x_{21} - x_{22} - x_{23} - x_{34}) + 12(300 - x_{31} - x_{32} - x_{33} - x_{34})$. And the shipping cost is $12x_{11} + 15x_{12} + 10x_{13} + 25x_{14} + 10x_{21} + 19x_{22} + 11x_{23} + 30x_{24} + 21x_{31} + 30x_{32} + 18x_{33} + 40x_{34}$. The sum of shipping cost and storage cost would be our objective function. And the constraints are listed above, together with the nonnegative constraint that all $x_{ij} \geq 0$.

2.4.6

Suppose we ship $x_{ij}$ units from Source $i$ to Destination $j$ for $(i, j) \neq (2, 1)$. $x_{21}$ is the number of units below 20 sent from Source 2 to Destination 1, and $y$ is the number of units above 20 sent from Source 2 to Destination 1.
minimize $8x_{11} + 17x_{12} + 19x_{13} + 21x_{22} + 22x_{23} + 10x_{21} + 13y$
subject to $x_{11} + x_{12} + x_{13} \leq 80$
$x_{21} + y + x_{22} + x_{23} \leq 80$
$x_{11} + x_{21} + y \geq 50$
$x_{12} + x_{22} \geq 50$
$x_{13} + x_{23} \geq 50$
x_{ij} \geq 0, x_{21} \leq 20, y \geq 0$

2.5.3
We assume that in the $i$-th month, the dealer buys $B_i$ units from the manufacturer, sells to the student $S_i$ units, and store $T_i$ units.

Maximize $(90S_1 + 110S_2 + 105S_3) - (60B_1 + 65B_2 + 68B_3) - 7(T_1 + T_2 + T_3)$
subject to $25 + B_1 = S_1 + T_1$
$T_1 + B_2 = S_2 + T_2$
$T_2 + B_3 = S_3 + T_3$
$0 \leq B_i \leq 65, 0 \leq S_i \leq 100, 0 \leq T_i \leq 45.$

2.6.9
Suppose in the $i$-th month, the shop produces $D_i$ units of differentials, and store $S_i$ units. Each month the shop uses $B_i$ hours below 400 hrs, and $O_i$ hours above 400 hrs.

minimize $3(12D_1 + 17D_2 + 25D_3 + 26D_4) + 18(B_1 + B_2 + B_3 + B_4)$
$+26(O_1 + O_2 + O_3 + O_4) + 10(S_1 + S_2 + S_3 + S_4)$
subject to $D_1 = 225 + S_1$
$S_1 + D_2 = 225 + S_2$
$S_2 + D_3 = 225 + S_3$
$S_3 + D_4 = 225 + S_4$
$2D_i \leq B_i + O_i$
$D_i \geq 0, S_i \geq 0, 0 \leq B_i \leq 400, 0 \leq O_i \leq 150.$

3.1.3
(e) First we introduce two new variables $x_3$ and $x_4$ to convert the inequalities into equalities:

Minimize $6x_1 + x_2$
subject to $-5x_1 + 8x_2 + x_3 = 80$
$x_1 + 2x_1 - x_4 = 4$
x_1 \leq 10, x_2, x_3, x_4 \geq 0.$
To replace \(x_1\) by a nonnegative variable. There are two ways: the first is to let \(x_1 = 10 - x_1'\), then \(x_1' = 10 - x_1 \geq 0\). Replace all the \(x_1\) by 10 − \(x_1'\) in the LP, we get:

\[
\begin{align*}
\text{Minimize} & \quad 6(10 - x_1') + x_2 \\
\text{subject to} & \quad -5(10 - x_1') + 8x_2 + x_3 = 80 \\
& \quad (10 - x_1') + 2x_1 - x_4 = 4 \\
& \quad x_1', x_2, x_3, x_4 \geq 0.
\end{align*}
\]

We could rearrange the terms so that the constant terms are on the right:

\[
\begin{align*}
\text{Minimize} & \quad 60 - 6x_1' + x_2 \\
\text{subject to} & \quad 5x_1' + 8x_2 + x_3 = 130 \\
& \quad -x_1' + 2x_1 - x_4 = -6 \\
& \quad x_1', x_2, x_3, x_4 \geq 0.
\end{align*}
\]

The other way is to first replace \(x_1\) by \(x_1' - x_1''\), with \(x_1', x_1'' \geq 0\). Then we introduce a new nonnegative variable \(x_5\), such that \(x_1' - x_1'' + x_5 = 10\). This also works but uses more variables than the previous one.

(f) We first change the objective function to minimizing \(-x_1 - 2x_2 - 4x_3\). For the absolute value, remember than \(|x| \leq y\) is equivalent to that both \(x \leq y\) and \(x \geq -y\) hold. So we can rewrite the LP as follows:

\[
\begin{align*}
\text{Minimize} & \quad -x_1 - 2x_2 - 4x_3 \\
\text{subject to} & \quad 4x_1 + 3x_2 - 7x_3 \leq x_1 + x_2 + x_3 \\
& \quad 4x_1 + 3x_2 - 7x_3 \geq -x_1 - x_2 - x_3 \\
& \quad x_1, x_2, x_3 \geq 0.
\end{align*}
\]

Simplifying the constraints we get

\[
\begin{align*}
\text{Minimize} & \quad -x_1 - 2x_2 - 4x_3 \\
\text{subject to} & \quad 3x_1 + 2x_2 - 8x_3 \leq 0 \\
& \quad 5x_1 + 4x_2 - 6x_3 \geq 0 \\
& \quad x_1, x_2, x_3 \geq 0.
\end{align*}
\]

Adding new variables \(x_4, x_5\), we get the standard form:

\[
\begin{align*}
\text{Minimize} & \quad -x_1 - 2x_2 - 4x_3 \\
\text{subject to} & \quad 3x_1 + 2x_2 - 8x_3 + x_4 = 0 \\
& \quad 5x_1 + 4x_2 - 6x_3 - x_5 = 0 \\
& \quad x_1, x_2, x_3, x_4, x_5 \geq 0.
\end{align*}
\]