Then let the functions $M(x,y)$, $N(x,y)$, $\frac{\partial}{\partial y}[M(x,y)]$, $\frac{\partial}{\partial x}[N(x,y)]$ be continuous in a "nice" region.

**THEN**

1st) $\frac{\partial}{\partial y}[M(x,y)] = \frac{\partial}{\partial x}(N(x,y))$

if and only if

$$[M(x,y)] + (N(x,y)) \frac{dy}{dx} = 0 \text{ is exact.}$$

2nd) D.E. is exact

if and only if

there is a function $\psi(x,y)$ such that

$$\frac{\partial}{\partial x} \psi(x,y) = [M(x,y)]$$

AND

$$\frac{\partial}{\partial y} \psi(x,y) = (N(x,y))$$
Given the D.E.

\[ [M(x,y)] + (N(x,y)) \frac{dy}{dx} = 0 \]

**Step 0:** Check if D.E. is exact, by computing

\[ \frac{\partial}{\partial y} [M(x,y)] = ? \]
\[ \frac{\partial}{\partial x} (N(x,y)) = ? \]

Are these the same?

If **NOT** the same I STOP! write "NOT exact."

If they are the same, go to Step 1.

**Step 1:** write "Since D.E. is exact, there is \( \psi(x,y) \) such that

\[ \frac{\partial}{\partial x} \psi(x,y) = [M(x,y)] \]
AND

\[ \frac{\partial}{\partial y} \psi(x,y) = (N(x,y)) \]"
Step 2:

\[ \psi(x, y) = \left( \int [M(x, y)] \, dx \right) + h(y) \]

Here we integrate as if the latter is a constant function of \( y \).

Step 3:

\[ (N(x, y)) = \frac{\partial}{\partial y} \left( \int [M(x, y)] \, dx + h(y) \right) \]

So

\[ \frac{d}{dy} (h(y)) = (N(x, y)) - \frac{\partial}{\partial y} \left( \int [M(x, y)] \, dx \right) \]

Step 4:

\[ h(y) = \int \left( (N(x, y)) - \frac{\partial}{\partial y} \left( \int [M(x, y)] \, dx \right) \right) dy \]

Step 5: use Step 2 & Step 4 and write

"A solution is given implicitly by"

\[ c = \left( \int [M_0(x, y)] \, dx \right) + \int \left( (N(x, y)) - \frac{\partial}{\partial y} \left( \int [M(x, y)] \, dx \right) \right) dy \]
Alternative Step 2:
\[ \psi(x, y) = \left( \int (N(x, y)) \, dy \right) + k(x) \]

Yet to be found function of \( x \)

Here we integrate as if the letter \( x \) is a constant.

Alternative Step 3:
\[ [M(x, y)] = \frac{2}{dx} \left( \int (N(x, y)) \, dy + k(x) \right) \]

So
\[ \frac{d}{dx} (k(x)) = [M(x, y)] - \frac{2}{dx} \left( \int (N(x, y)) \, dy \right) \]

Alternative Step 4:
\[ k(x) = \int \left( [M(x, y)] - \frac{2}{dx} \left( \int (N(x, y)) \, dy \right) \right) \, dx \]

Alternative Step 5: use Alternative Step 2 & Alternative Step 4 and write
"A solution is given implicitly by"
\[ c = \left( \int (N(x, y)) \, dy \right) + \int \left( [M(x, y)] - \frac{2}{dx} \left( \int (N(x, y)) \, dy \right) \right) \, dx \]