Sequences and Series Problems (due November 9)

1. Is π the limit of an infinite sequence of real numbers of the form $\sqrt[3]{n} - \sqrt[3]{m}$ with n, m nonnegative integers?

2. Evaluate the infinite sum
$$\sum_{n=1}^{\infty} \frac{n^3}{3^n}.$$

3. Suppose that a sequence $a_1, a_2, a_3, \ldots$ of positive real numbers satisfies $a_n \le a_{2n} + a_{2n+1}$ for all $n \ge 1$. Prove that the series $\sum_{n=1}^{\infty} a_n$ diverges.

4. Let N be the set of all positive integers which do not contain the digit 9 in their decimal representation. Prove that
$$\sum_{a \in N} \frac{1}{a} < 80.$$

5. Let P be the set of all positive integers of the form a^b with $a, b \ge 2$ integers. Prove that
$$\sum_{q \in P} \frac{1}{q-1} = 1.$$

6. Define a sequence a_n by the power series identity
$$\frac{1}{1 - 2x - x^2} = \sum_{n=0}^{\infty} a_n x^n \ .$$
Prove that $a_n^2 + a_{n+1}^2 = a_{2n+2}$ for all $n \ge 0$.