Define a sequence $v_n$ by

$$v_n = (n - 1)(n - 3) \cdots (4)(2)$$

if $n$ is odd

and

$$v_n = (n - 1)(n - 3) \cdots (3)(1)$$

if $n$ is even.

It suffices to prove that $u_n = v_n$ for all $n \geq 2$. Now

$$v_{n+3}v_n = (n+2)(n)(n-1)!$$

and $v_{n+2}v_{n+1} = (n+1)!$, and so $v_{n+3}v_n - v_{n+2}v_{n+1} = n!$. Since we can check that $u_n = v_n$ for $n = 2, 3, 4$, and $u_n$ and $v_n$ satisfy the same recurrence, it follows by induction that $u_n = v_n$ for all $n \geq 2$, as desired.