6. Divide the rectangle in the $xy$-plane $0 \leq x \leq 1$, $0 \leq y \leq e$ into two regions $A$, the area above $y = e^{x^2}$, and $B$, the area below $y = e^{x^2}$. We have $\text{area}(A) + \text{area}(B) = e$ and $\text{area}(B) = \int_0^1 e^{x^2} \, dx$. Also, by interchanging the roles of $x$ and $y$, so $y = e^{x^2}$ becomes $x = \sqrt{\ln y}$, we see that $\text{area}(A) = \int_1^e \sqrt{\ln y} \, dy$. Now make the substitution $x = (y - 1)/(e - 1)$, so $y = 1 + ex - x$, we see that this last integral is $\int_0^1 (e - 1) \ln(1 + ex - x) \, dx$. We conclude that $\int_0^1 ((e - 1) \sqrt{\ln(1 + ex - x)} + e^{x^2}) \, dx = e$.

7. Suppose $AA'x = 0$, where $x$ is a column vector with 5 components. Then $x'AA'x = 0$, hence $(A'x)'(A'x) = 0$. Since all the entries of $A'x$ are real numbers, we deduce that $A'x = 0$. Thus $A'$ and $AA'$ have the same null space. By hypothesis $\text{rank}(A) = 5$, hence $\text{rank}(A') = 5$ and we deduce that the null space of $A'$ is 0. Therefore $AA'$ has zero null space and we deduce that $\text{rank}(AA') = 5$. This means that every $5 \times 1$ matrix can be written in the form $AA'v$. 