

# Sample BMO Problems

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Again, these are BMO problems from Gardiner's book.

**1975, Problem 3:** Given a point  $O$  in the plane, suppose  $S$  is a set of seven points at distance at most one from  $O$ . Suppose that for every pair of points in  $S$ , their distance is at least one. Show that  $O \in S$ . (Hint: use the pigeon-hole principle.)

**1980, Problem 1:** Prove that the equation  $x^n + y^n = z^n$ , where  $n$  is an integer  $> 1$ , has no solution in integers  $x$ ,  $y$ , and  $z$  such that  $0 < x \leq n$  and  $0 < y \leq n$ .

**1981, Problem 4:** Consider  $n$  points in three-dimensional space. Let  $S$  be the set of all tetrahedra formed by choosing four of the points. Suppose a plane  $P$  does not pass through any of the points. Show that  $P$  intersects at most  $n^2(n-2)^2/64$  of the tetrahedra of  $S$  in a quadrilateral cross-section.

**1981, Problem 5:** Find the smallest possible value of  $|12^m - 5^n|$ , where  $m$  and  $n$  are positive integers.