

# Sample Problems

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Again, these are relatively simple problems.

**USAMO 1978:** Nine mathematicians meet at an international conference and discover that among any three of them, at least two speak a common language. If each of the mathematicians can speak at most three languages, prove that there are at least three of the mathematicians who speak the same language.

**USAMO 1994:** Let  $a, b$  be odd positive integers. Define the sequence  $(f_n)$  by putting  $f_1 = a$ ,  $f_2 = b$ , and for  $n \geq 3$  let  $f_n$  be the greatest odd divisor of  $f_{n-1} + f_{n-2}$ . Show that  $f_n$  is a constant for  $n$  sufficiently large, and determine this constant as a function of  $a$  and  $b$ .

**BKM 16:**  $EFGH$  is a square inscribed in the quadrilateral  $ABCD$ . If  $\overline{EB} = \overline{FC} = \overline{GD} = \overline{HA}$ , prove that  $ABCD$  is also a square.