Name

Mathematics 107
Exam #3

Spring 2011
Q1. (4 points) Suppose $X$ is a random variable with mean $\mu = 50$ and variance $\sigma^2 = 49$. A random sample of size $n = 38$ is selected from this population.

(a) Find the approximate distribution of $\bar{X}$. Why is the Central Limit Theorem necessary here?

$\bar{X} \sim N(50, 49/38)$ (large sample)

The CLT is necessary because the shape of the underlying distribution is not known.

(b) Find $P(\bar{X} < 49)$

$$P(\bar{X} < 49) = P(Z \leq \frac{49 - 50}{\sqrt{49/38}}) = P(Z \leq -0.88)$$

$$= 0.1894$$

(c) Find $P(\bar{X} \geq 52)$

$$P(\bar{X} \geq 52) = P(Z \geq \frac{52 - 50}{\sqrt{49/38}}) = P(Z \geq 1.76)$$

$$= 1 - P(Z < 1.76) = 1 - 0.9608 = 0.0392$$

(d) Find $P(49.5 \leq \bar{X} \leq 51.5)$

$$P(49.5 \leq \bar{X} \leq 51.5) = P\left(\frac{49.5 - 50}{\sqrt{49/38}} \leq Z \leq \frac{51.5 - 50}{\sqrt{49/38}}\right)$$

$$= P(-0.44 \leq Z \leq 0.32) = P(Z \leq 0.32) - P(Z \leq -0.44)$$

$$= 0.9066 - 0.3300 = 0.5766$$
Q2. (2 points) Suppose a random sample of size \( n = 200 \) is obtained from a population with probability of success \( p = 0.40 \). Find each of the following probabilities.

(a) Find \( P(\hat{p} < 0.37) \)

\[
\hat{p} \sim N(0.4, 0.0012)
\]

\[
P(\hat{p} < 0.37) = P\left( z < \frac{0.37 - 0.4}{\sqrt{0.0012}} \right)
\]

\[
= P\left( z < -0.87 \right) = 0.1922
\]

(b) Find \( P(0.38 \leq \hat{p} \leq 0.42) \)

\[
P(0.38 \leq \hat{p} \leq 0.42) = P\left( \frac{0.38 - 0.4}{\sqrt{0.0012}} \leq z \leq \frac{0.42 - 0.4}{\sqrt{0.0012}} \right)
\]

\[
= P(-0.58 \leq z \leq 0.58)
\]

\[
= P( z \leq 0.58) - P( z \leq -0.58)
\]

\[
= 0.7190 - 0.2810
\]

\[
= 0.4380
\]
Q3. (2 points) In each of the following problems, the sample mean, the sample size, the population standard deviation, and the confidence level are given. Assume the underlying population is normally distributed. Find the associated confidence interval for the population mean.

(a) $\bar{x} = 6322, \quad n = 17, \quad \sigma = 225, \quad 90\%$

$$\bar{x} \pm z_{0.05} \frac{\sigma}{\sqrt{n}} = 6322 \pm (1.6449) \frac{225}{\sqrt{17}}$$

$$= (6232.24, 6411.76)$$

(b) $\bar{x} = -45.78, \quad n = 9, \quad \sigma = 12.35, \quad 80\%$

$$\bar{x} \pm z_{0.10} \frac{\sigma}{\sqrt{n}} = -45.78 \pm (1.2816) \frac{12.35}{\sqrt{9}}$$

$$= (-51.06, -40.50)$$
Q4. (2 points) In each of the following problems, the population standard deviation, the bound on the error of estimation, and the confidence level are given. Find a value for the sample size \( n \) necessary to satisfy these requirements.

(a) \( \sigma = 10.77, \ B = 5, \ 99\% \)

\[
N = \left[ \frac{(10.77)(2.576)}{5} \right]^2 = 30.78
\]

\( \Rightarrow \ n \geq 31 \)

(b) \( \sigma = 0.55, \ B = 0.001, \ 98\% \)

\[
N = \left[ \frac{(0.55)(2.326)}{0.001} \right]^2 = \frac{637}{0.03069}
\]

\( \Rightarrow \ n \geq 16371.021 \)
Q5. (2 point) In many areas, newspaper carriers deliver morning papers using automobiles because the routes are too long to walk. In a random sample of 28 carriers who use their automobiles, the sample mean route length was $\bar{x} = 16.7$ with the sample standard deviation $s = 3.4$. If the distribution of rout lengths is normal, find 95% confidence interval for the true mean route length of newspaper carriers who use their automobiles.

$$
\bar{x} \pm t_{0.025} \frac{s}{\sqrt{n}} = 16.7 \pm (2.0518) \frac{3.4}{\sqrt{28}}
$$

$$
= (15.3816, 18.0184)
$$
Q6. (4 points) Residential mailboxes in Des Moines, Iowa, should be installed such that the bottom of the mailbox is 42 inches above the ground. The rule is designed for safety and accommodate short mail carriers. A random sample of 75 mailboxes was selected. The height of each was carefully measured, and the sample mean was 43.22 inches. Assume $\sigma = 7.6$ inches and use $\alpha = 0.05$. Is there any evidence to suggest that the true mean height of mailboxes in Des Moines is different from 42 inches?

(i) Hypothesis

$$H_0: \mu = 42$$

$$H_1: \mu \neq 42$$

(ii) The test statistic (TS)

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{43.22 - 42}{7.6/\sqrt{75}} = 1.3902$$

(iii) The rejection region (RR)

$$\alpha = 0.05$$

$$\frac{\alpha}{2} = 0.025$$

(iv) Decision

Do not reject $H_0$. There is no evidence to suggest that the true mean height of mailboxes in Des Moines is different from 42 inches.
Q7. (2 points) Consider a hypothesis test concerning population mean with $\sigma$ known and $n$ large. For each alternative hypothesis, value of the test statistic, and significance level, find the $p$ value and determine whether $H_0$ is rejected or not rejected.

(a) $H_a : \mu \neq 1200, \quad z = 1.75, \quad \alpha = 0.10$

$$p = 0.0801$$

Reject $H_0$ since $\alpha > p$

(b) $H_a : \mu < 52.68, \quad z = -1.16, \quad \alpha = 0.025$

$$p = 0.1230$$

Do not reject $H_0$ since $\alpha < p$
Q8. (4 points) A 42-inch, large screen, plasma TV is designed to consume only 350 watts and therefore produce less heat. In order to check this specification, a random sample of seven TVs was obtained and the power consumption was measured for each. The sample mean was $\bar{x} = 353.8$ watts and the sample standard deviation $s = 5.6$ watts. Is there any evidence to suggest that the mean power consumption for this model plasma TV does not meet the design specification? Assume normality and use $\alpha = 0.05$.

(i) Hypothesis

\[ H_0: \mu = 350 \]
\[ H_a: \mu > 350 \]

(ii) The test statistic (TS)

\[ T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{353.8 - 350}{5.6/\sqrt{7}} = 1.7953 \]

(iii) The rejection region (RR)

\[ \alpha = 0.05 \]
\[ df = n - 1 = 7 - 1 = 6 \]
\[ a) T \geq 1.9432 \]
\[ b) \frac{s}{\bar{X}} = 0.05 \]
\[ T \geq 2.4469 \]

(iv) Decision

Do not reject $H_0$. There is no evidence that the mean power consumption for this model plasma TV fails to meet the designed specifications.
Q9. (4 points) A local planning board must consider whether to require all new housing projects with five or more apartments to designate some of the units as rent-controlled. A random sample of 100 apartments in Cheyenne, Wyoming was obtained, and 12 were found rent-controlled. Is there any evidence to suggest that the proportion of rent-controlled apartments is less than 10%? Use \( \alpha = 0.09 \).

(i) Hypothesis

\[ H_0: \ p = 0.10 \]
\[ H_a: \ p < 0.10 \]

(ii) The test statistic (TS)

\[ Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.12 - 0.10}{\sqrt{\frac{0.10(0.90)}{100}}} = 0.6667 \]

(iii) The rejection region (RR)

\( \alpha = 0.09 \)

\( Z < -1.34 \)

(iv) Decision

Do not reject \( H_0 \). There is no evidence to suggest that the proportion of rent-controlled apartments is less than 0.10.
Q10. (4 points) Bull riding has become a very popular rodeo sport - the action is fast and dangerous. The variance in bull-riding time tends to be large, approximately 22.5 seconds. A random sample of 37 bull rides was selected over the entire rodeo season. The sample variance in riding times was $s^2 = 15.6$ seconds. Smaller variability in bull-riding times translates to a more monotonous, unexciting rodeo. Is there any evidence that bull riding has become less exciting? Assume normality and use $\alpha = 0.05$.

(i) Hypothesis

\[ H_0 : \sigma^2 = 22.5 \]
\[ H_a : \sigma^2 < 22.5 \]

(ii) The test statistic (TS)

\[ \chi^2 = \frac{(n-1) s^2}{\sigma_0^2} = \frac{36 \cdot (15.6)}{22.5} = 24.96 \]

(iii) The rejection region (RR)

\[ \alpha = 0.05 \quad 1 - \alpha = 0.95 \]
\[ df = 36 \]
\[ \chi^2_{0.95, 36} = 23.2686 \]

(iv) Decision

Do not reject $H_0$. There is no evidence that bull riding has become less exciting.