Optimization Problems

STRATEGY:

1. Identify what needs to be maximized/minimized
2. Draw a diagram (if necessary)
3. Assign notation to the value we want to optimize (say, Q) and all other quantities
4. Express Q in terms of all other variables (say x, y, z, w, l, etc.)
5. Find relations between the other variables and use them to express Q in terms of **exactly one** variable, say x (i.e. $Q = f(x)$)
6. Find the absolute maximum/minimum value of the function

Note: If the domain of the function we are optimizing is not a closed interval and there is only one critical number in the domain, you must check that the critical number actually yields the desired maximum or minimum:

The First Derivative Test for Absolute Extrema:

If c is the only critical number of a continuous function $f(x)$ and:

1. if $f'x) > 0$ for all $x < c$ and $f'(x) < 0$ for all $x > c$, then $f(c)$ is the absolute maximum value of f.
2. if $f'x) < 0$ for all $x < c$ and $f'(x) > 0$ for all $x > c$, then $f(c)$ is the absolute minimum value of f.

1. The sum of two positive numbers is 16. What is the smallest possible value of the sum of their squares?

   - We want to minimize/maximize _____.
   - Define variables.
   - Find a relation for what we want to optimize.
   - Find a relation between the other variables and use it to write the optimization function in terms of one variable.
   - Find the absolute minimum/maximum value. (Be sure to check that it actually yields a minimum or maximum!)

2. A farmer has 500 feet of fencing and wants to build a rectangular pen with three parallel partitions. Find the dimensions of the pen that will maximize its total area.

   - We want to minimize/maximize _____.
   - Draw and label a diagram.
   - Find a relation for what we want to optimize.
   - Find a relation between the other variables and use it to write the optimization function in terms of one variable.
   - Find the absolute minimum/maximum value. (Be sure to check that it actually yields a minimum or maximum!)

3. A box with a square base and open top must have a volume of 32 cm^3. Find the dimensions of the box that minimize the amount of material used.

   - We want to minimize/maximize _____.
   - Draw and label a diagram.
   - Find a relation for what we want to optimize.
   - Find a relation between the other variables and use it to write the optimization function in terms of one variable.
   - Find the absolute minimum/maximum value. (Be sure to check that it actually yields a minimum or maximum!)

4. Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(3, 0)$.

   - We want to minimize/maximize _____.
   - Draw and label a diagram.
   - Find a relation for what we want to optimize.
   - Find a relation between the other variables and use it to write the optimization function in terms of one variable.
   - Find the absolute minimum/maximum value.