Math 515
Fall, 2008

Homework 4, due Friday, November 14.

1. Consider the 20×20 matrix,

$$A = \begin{bmatrix} 20 & 20 & & & \\ & 19 & 20 & & \\ & & \ddots & \ddots & \\ & & & 2 & 20 \\ & & & & 1 \end{bmatrix}$$

   (a) What are the eigenvalues of this matrix? Explain.

   (b) Use the MATLAB `eig` function to compute the eigenvalues of A. Do the computed results match what you found in part (a)?

   (c) Use the MATLAB `condeig` function to compute the condition number of the eigenvalues. What do these results tell you?

   (d) Now perturb the (20,1) entry of A by $\varepsilon = 10^{-10}$. Use the MATLAB `eig` function to compute the eigenvalues of the perturbed matrix. Are the results as you expected? Explain?

2. Consider the $n \times n$ matrix

$$A = \begin{bmatrix} n & (n-1) & (n-2) & \cdots & 3 & 2 & 1 \\ (n-1) & (n-1) & (n-2) & \cdots & 3 & 2 & 1 \\ & (n-2) & (n-2) & \cdots & 3 & 2 & 1 \\ & & \ddots & \ddots & \vdots & \vdots & \vdots \\ & & & & 2 & 2 & 1 \\ & & & & & 1 & 1 \end{bmatrix}$$

   Using the MATLAB functions `eig` and `condeig` investigate the conditioning of the eigenvalues of A for various values of n. Explain what you observe, and give some examples to illustrate your findings.