1. Consider the $20 \times 20$ matrix, 

$$A = \begin{bmatrix} 20 & 20 \\ 19 & 20 \\ & \ddots & \ddots \\ & 2 & 20 \\ & & 1 \end{bmatrix}$$

(a) What are the eigenvalues of this matrix? Explain.

(b) Use the MATLAB \texttt{eig} function to compute the eigenvalues of $A$. Do the computed results match what you found in part (a)?

(c) Use the MATLAB \texttt{condeig} function to compute the condition number of the eigenvalues. What do these results tell you?

(d) Now perturb the $(20,1)$ entry of $A$ by $\varepsilon = 10^{-10}$. Use the MATLAB \texttt{eig} function to compute the eigenvalues of the perturbed matrix. Are the results as you expected? Explain?

2. Consider the $n \times n$ matrix 

$$A = \begin{bmatrix} n & (n-1) & (n-2) & \cdots & 3 & 2 & 1 \\ (n-1) & (n-1) & (n-2) & \cdots & 3 & 2 & 1 \\ (n-2) & (n-2) & \cdots & 3 & 2 & 1 \\ & \ddots & \ddots & \ddots & \ddots & \ddots \\ & 2 & 2 & 1 \\ & & & 1 & 1 \end{bmatrix}$$

Using the MATLAB functions \texttt{eig} and \texttt{condeig} investigate the conditioning of the eigenvalues of $A$ for various values of $n$. Explain what you observe, and give some examples to illustrate your findings.