Math 351
Spring, 2008

Practice Problems for Chapter 1

1. Verify that each of the given functions satisfies the heat equation (with no external sources):

$$u_t - k u_{xx} = 0$$

for $0 < x < \pi$, and the accompanying boundary and intitial conditions.

   (a) $u(x,t) = e^{-kt}\sin(x)\,, \quad u(x,0) = \sin(x)\,, \quad u(0,t) = u(\pi,t) = 0.$

   (b) $u(x,t) = e^{-kt}\cos(x)\,, \quad u(x,0) = \cos(x)\,, \quad u_x(0,t) = u_x(\pi,t) = 0.$

   (c) $u(x,t) = 0.5 + 0.5e^{-4kt}\cos(2x)\,, \quad u(x,0) = \cos^2(x)\,, \quad u_x(0,t) = u_x(\pi,t) = 0$

2. (a) Show that the function

$$u(x,t) = e^{-k\theta^2 t}\sin(\theta x)$$

   is a solution of the homogeneous (i.e., no external sources) heat equation

$$\frac{\partial u}{\partial t} - k\frac{\partial^2 u}{\partial x^2} = 0\,, \quad 0 < x < L\,, \quad \text{for all } t.$$

   (b) What values of θ will cause u to also satisfy homogeneous Dirichlet boundary conditions at $x = 0$ and $x = L$? Recall that homogeneous Dirichlet boundary conditions mean $u(0,t) = u(L,t) = 0$.

3. From the Haberman text, do exercise 1.4.1, page 19. Note that in this exercise, the equilibrium differential equation is

$$\frac{d^2u}{dx^2} = -\frac{Q}{K_0}\,.$$

4. Compute gradient vector ∇u and the Laplacian, $\nabla^2 u$, for the following functions.

   (a) $u(x,y) = 3x^2y^2 + 2e^{xy}$

   (b) $u(x,y,z) = 2x^2 + y^2 + z^2$

   (c) $u(x,y) = \ln(x^2 + 3xy + 2y^2)$

   (d) $u(x,y,z) = y^2z^2(1 + \sin^2 x) + (y+1)^2(z+3)^2$