Math 351
Spring, 2008

Partial solutions for: Practice Problems for Chapter 1

1. Substitute into the PDE. Note that you have to check to make sure the boundary and initial conditions are satisfied too.

2. (a) Verify by substituting into the PDE.
   (b) $\theta = \pm\dfrac{n\pi}{L}$, $n = 0, 1, 2, \ldots$

3. (a) $u(x) = \frac{T}{L}x$
   (b) $u(x) = -\frac{T}{L}x + T$
   (c) $u(x) = T$
   (d) $u(x) = \alpha x + T$
   (e) $u(x) = -\dfrac{1}{2}x^2 + \left(\dfrac{T_2 - T_1 + \frac{1}{2}L^2}{L}\right)x + T_1$
   (f) $u(x) = -\frac{1}{12}x^4 + \frac{1}{3}L^3 x + T$
   (g) $u(x) = -\dfrac{T}{L+1}x + T$
   (h) $u(x) = \alpha x + (T + \alpha)$

4. (a) $\nabla u = \begin{pmatrix} u_x \\ u_y \end{pmatrix} = \begin{pmatrix} 6xy^2 + 2ye^{xy} \\ 6x^2y + 2xe^{xy} \end{pmatrix}, \quad \nabla^2 u = u_{xx} + u_{yy} = 2(x^2 + y^2)(e^{xy} + 3)$
   (b) $\nabla u = \begin{pmatrix} 4x \\ 2y \\ 2z \end{pmatrix}, \quad \nabla^2 u = 8$
   (c) $\nabla u = \begin{pmatrix} (2x + 3y)/(x^2 + 3xy + 2y^2) \\ (3x + 4y)/x^2 + 3xy + 2y^2) \end{pmatrix}, \quad \nabla^2 u = \dfrac{-7x^2 - 18xy - 13y^2}{(x^2 + 3xy + 2y^2)^2}$
   (d) $\nabla u = \begin{pmatrix} 2y^2z^2 \sin x \cos x \\ 2yz^2(1 + \sin^2 x) + 2(y+1)(z+3)^2 \\ 2y^2z(1 + \sin^2 x) + 2(y+1)^2(z+3) \end{pmatrix},$

   $$\nabla^2 u = 2y^2z^2(\cos^2 x - \sin^2 x) + 2z^2(1 + \sin^2 x) + 2(z+3)^2 + 2y^2(1 + \sin^2 x) + 2(y+1)^2$$