1. For each of the following, state the order of the ODE and whether it is linear or nonlinear.

(a) $y'/y = x^2 + x$
(b) $\sin(y') = 5y$
(c) $y'' = 3x^2$
(d) $y''' = y^3$
(e) $y'' + xy = \cos(y'')$
(f) $(1 + x^2)y' = (1 + y)^2$
(g) $y' + y + y^2 = x + e^x$
(h) $x^2y'' + x^{1/2}(y')^3 + y = e^x$

2. Solve the ODE initial value problem:

$$\frac{dy}{dx} = x + 1, \quad y(1) = 2$$

3. For each of the following, state the order of the PDE and whether it is linear or nonlinear.

(a) $\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = (1 + t) \sin(x)$
(b) $\frac{\partial w}{\partial t} + w \frac{\partial w}{\partial x} = 0$
(c) $\frac{\partial u}{\partial t} - \frac{\partial}{\partial x} \left( 2u \frac{\partial u}{\partial x} \right) = 0$
(d) $\frac{\partial u}{\partial t} - \frac{\partial}{\partial x} \left( 2x \frac{\partial u}{\partial x} \right) = 0$

4. (a) Determine if $u(x, t) = t \sin(x)$ is a solution of problem 3(a).
   (b) Determine if $w(x, t) = t(1 - x)$ is a solution of problem 3(b).

5. Consider the PDE:

$$4 \frac{\partial u}{\partial x} + 3 \frac{\partial u}{\partial y} + u = 0$$

Show that all of the following functions are solutions of this PDE.

(a) $e^{-x^4}(9x^2 - 24xy + 16y^2)$.
(b) $e^{-x^4}e^{3x^4 - 4y}$.
(c) $e^{-x^4}f(3x - 4y)$, where $f(z)$ is any differentiable function of $z$. 
6. Determine if the following PDEs are parabolic, elliptic, or hyperbolic.

   (a) $u_{xx} + 2 \cos(x)u_{xy} - \sin^2(x)u_{yy} - \sin(x)u_y = 0$.
   (b) $u_{xx} + 4u_{xy} + 4u_{yy} = 0$.
   (c) $9u_{xx} + 12u_{xy} + 4u_{yy} + u_x = 0$.
   (d) $u_{xx} + 2u_{xy} + 3u_{yy} + 4u = 0$.
   (e) $u_{xx} - 8u_{xy} + 2u_{yy} + xu_x - yu_y = 0$.

7. Consider the PDE

   \[ yu_{xx} + u_{yy} = 0 \]

   Determine for which points $(x, y)$ is the PDE hyperbolic, which points $(x, y)$ is the PDE parabolic, and which points $(x, y)$ is the PDE elliptic.

8. Determine where (in the $x, y$-plane) the following PDE is hyperbolic, elliptic, and parabolic.

   \[ x^2y \frac{\partial^2 u}{\partial x^2} + xy \frac{\partial^2 u}{\partial x \partial y} - y^2 \frac{\partial^2 u}{\partial y^2} = 0 \]