1. Consider the function

\[ f(x) = \begin{cases} 
(x + 1)^3 - 1, & -2 \leq x \leq 0 \\
 x^2 + 1, & 0 < x \leq 2 
\end{cases} \]

(a) Sketch the graph of \( f(x) \) on the interval \(-2 \leq x \leq 2\).

(b) Sketch the graph of the Fourier series of \( f(x) \). (Sketch at least three periods).
2. For the following functions:

- Sketch the graph of \( f(x) \) for \(-L \leq x \leq L\)
- Sketch the graph of the Fourier series for \( f(x) \) (sketch at least 3 periods).
- Determine the Fourier coefficients.

(a) \( f(x) = x \)
(b) \( f(x) = e^{-x} \)
(c) \( f(x) = \sin \left( \frac{\pi x}{L} \right) \)
(d) \( f(x) = \begin{cases} 0 & x < 0 \\ x & x > 0 \end{cases} \)
(e) \( f(x) = \begin{cases} 0 & x < 0 \\ 1 & x > 0 \end{cases} \)

3. For the following functions, sketch the Fourier sine series of \( f(x) \) and determine its Fourier coefficients.

(a) \( f(x) = \cos \left( \frac{\pi x}{L} \right) \)
(b) \( f(x) = \begin{cases} 0 & x < \frac{L}{2} \\ x & x > \frac{L}{2} \end{cases} \)
(c) \( f(x) = \begin{cases} 1 & x < \frac{L}{2} \\ 0 & x > \frac{L}{2} \end{cases} \)

4. For the following functions, sketch the Fourier cosine series of \( f(x) \) and determine its Fourier coefficients.

(a) \( f(x) = x^2 \)
(b) \( f(x) = \begin{cases} 0 & x < \frac{L}{2} \\ x & x > \frac{L}{2} \end{cases} \)

5. Show that \( e^x \) is the sum of an even and an odd function.

6. Determine the even and odd parts of

\[
    f(x) = \begin{cases} 
        x^2 & x < 0 \\
        e^{-x} & x > 0 
    \end{cases}
\]