Partial solutions for Practice Problems 1

1. (a) 1st order, linear
   (b) 1st order, nonlinear
   (c) 2nd order, linear
   (d) 3rd order, nonlinear
   (e) 2nd order, nonlinear
   (f) 1st order, nonlinear
   (g) 1st order, nonlinear
   (h) 2nd order, nonlinear

2. $y(x) = (x + 1)^2/2$

3. (a) 2nd order, linear
   (b) 1st order, nonlinear
   (c) 2nd order, nonlinear
   (d) 2nd order, linear

4. (a) $u(x, t) = t \sin(x)$ is a solution of problem 3(a). Verify by substituting into the PDE.
   (b) $w(x, t) = t(1 - x)$ is NOT a solution of problem 3(b).

5. Verify by substituting functions into the PDE.

6. Put PDE in the form
   \[ A u_{xx} + 2B u_{xy} + C u_{yy} + D u_x + E u_y + F u + G = 0 \]
   and look at discriminant $B^2 - AC$ (> 0 means hyperbolic, = 0 means parabolic, < 0 means elliptic).
   (a) hyperbolic
   (b) parabolic
   (c) parabolic
   (d) elliptic
   (e) hyperbolic

7. Hyperbolic for all $x$ and all $y < 0$.
   Parabolic for all $x$ and $y = 0$.
   Elliptic for all $x$ and $y > 0$.

8. Hyperbolic if $x \neq 0$ and $y \neq 0$ and $y > -1/4$.
   Parabolic if $x = 0$ or $y = 0$ or $y = -1/4$.
   Elliptic if $x \neq 0$ and $y \neq 0$ and $y < -1/4$. 
