Related rates

Air is being pumped into spherical balloon and the rate of volume increase is 100 cm$^3$/s. How fast is radius of balloon increasing when diameter is 50 cm?

\[ \frac{dV}{dt} = 100 \text{ cm}^3/\text{s} \]

\[ V = \frac{4}{3} \pi r^3 \]

\[ \frac{dV}{dt} = \frac{4}{3} \pi 3r^2 \frac{dr}{dt} \]

\[ 100 = 4\pi r^2 \frac{dr}{dt} \]

\[ \frac{dr}{dt} = \frac{100}{4\pi (25)^2} = \frac{1}{25\pi} \]

\[ \text{Ans} = \frac{1}{25\pi} \text{ cm/s} \]
2) A 10 ft long ladder rests against a wall. The bottom of the ladder slides away at a rate of 1 ft/s. How fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

\[ x^2 + y^2 = 100 \]

\[ \frac{dx}{dt} = 1 \]

\[ \frac{dy}{dt} = ? \]

\[ 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \]

\[ \frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt} = -\frac{x}{y} \]

If \( x = 6 \), \( y = 8 \)

\[ \frac{dy}{dt} = -\frac{6}{8} = -\frac{3}{4} \text{ ft/s} \]

This means the top of the ladder is sliding down the wall.
3) Inverted conical core, base radius \( r = 2 \text{ m} \), height \( h = 4 \text{ m} \)

Water is pumped into it at a rate of \( 2 \text{ m}^3/\text{min} \).

Rate at which water level is rising when water is \( 3 \text{ m} \) deep:

\[
V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \left( \frac{h}{2} \right)^2 h
\]

\[
\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \frac{dh}{dt}
\]

\[
= \frac{\pi}{4} h^2 \frac{dh}{dt}
\]

\[
\therefore \quad \frac{dh}{dt} = \frac{\frac{dV}{dt}}{\frac{\pi}{4} h^2} = \frac{2}{\frac{8}{9\pi}} = \frac{9\pi}{8} \text{ m/s}
\]

4) Car A travelling west \( 50 \text{ m/h} \)

Car B travelling north \( 60 \text{ m/h} \)

At what rate are cars approaching each other when \( A = 0.3 \text{ mi} \), \( B = 0.4 \text{ mi} \) from intersection?
\[ \frac{dx}{dt} = -50 \quad \frac{dz}{dt} = ? \]
\[ \frac{dy}{dt} = -60 \]

\[ z^2 = x^2 + y^2 \]

\[ 2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt} \]

\[ \begin{align*}
2z &= 0.3 \\
0.4 &= 0.5
\end{align*} \]

\[
\frac{dz}{dt} = \frac{1}{0.5} \left( 0.3(-50) + 0.4(-60) \right)
\]

\[ = -78 \]
A man walks along a straight path at speed 4 ft/s.

Searchlight on ground 20 ft from path, focused on man.

At what rate is searchlight rotating when man is 15 ft
from pt on path closest to searchlight.

\[ \theta = \frac{x}{20} \]

\[ x = 20 \tan \theta \]

\[ \frac{dx}{dt} = 20 \sec^2 \theta \frac{d\theta}{dt} \]

\[ \frac{4}{20 \sec^2 \theta} = \frac{d\theta}{dt} \]

\[ \cos^2 \theta = \frac{16}{25} = 0.64 \]