Recap

1. What is a function, domain, range

   ex: Find domain

   \[ \begin{align*}
   \text{(1)} & \quad \sqrt{x+2} & \quad [\text{ } -2, \infty) \\
   \text{(2)} & \quad \frac{1}{x^2-x} & \quad \mathbb{R} \setminus [0, 1]
   \end{align*} \]

   Find range

   \[ \begin{align*}
   \text{(1)} & \quad f(x) = \sin x & \quad [-1, 1] \\
   \text{(2)} & \quad f(x) = \sin^2 x & \quad [0, 1] \\
   \text{(3)} & \quad f(x) = x^2 + 1 & \quad [0, \infty) \\
   \text{(4)} & \quad f(x) = x^2 - 1 & \quad [-1, \infty)
   \end{align*} \]

2. How to check if a function is well a function

   Vertical line test:

   ![Graph showing a vertical line intersecting the graph at more than one point]

   If a vertical line intersects graph at more than 1 point, it is NOT a function

   Think about

   Horizontal line test: what does it say?

   Introduce \( 1-1 \), onto functions

   injective, surjective
3. **Sin, cos graph and formulae**

\[ \sin \left( \frac{\pi}{2} + a \right), \cos \left( \frac{\pi}{2} + a \right), \sin (\pi + a) \text{ etc.} \]

4. **Line equation**

\[ mx + c = y \]

\[ c = x \]

**Slope**

\[ y = mx + c \quad m \quad \text{or} \quad \frac{\Delta y}{\Delta x} = \frac{y_1 - y_0}{x_1 - x_0} \]

*What is the slope of* \( 3x + 5y - 10 = 0 \)

\[ 3x - \pi = 0 \]

*What is the slope of the line* \( \frac{2-0}{3-0} = \frac{2}{3} \)

5. **Find the equation of a line with slope 2 and passing through (1,3)**

**Step 1:**

\[ y = mx + c \quad \text{Find } m, c \]

**Step 2:**

\[ m = 2 \]

\[ y = 2x + c \]

\[ y = 2x + 1 \]

**Step 3:**

\[ x = 1, \; y = 3 \]

\[ 3 = 2 \times 1 + c \quad \text{so} \]

\[ 3 = 2 + c \]

\[ \therefore c = 1 \]
(6) Find a cubic \( f \) so that

\[
\begin{align*}
-f(1) &= 6 \\
f(-1) &= 0 \\
f(2) &= 0 \\
f(3) &= 0
\end{align*}
\]

\[
f = a(x+1)(x-2)(x-3)
\]

Plug in 1

\[
f(1) = a(2)(-1)(1) = -2a
\]

so \(-2a = 6\), \(a = 3\)

\[
-3(x+1)(x-2)x
\]

(7) Solve quadratic equation

\[
\begin{align*}
(1) & \quad x^2 + 3x + 2 = 0 \\
& \quad (-2, -1) \\
(2) & \quad x^2 - x - 20 = 0 \\
& \quad (5, -4)
\end{align*}
\]

(8) Composition of functions

\[
\begin{align*}
\text{Composition of functions} & \quad f(x) \quad g(f(x)) \quad \text{"} g \circ f \text{"} \\
\end{align*}
\]

\[
\begin{align*}
\text{Composition of functions} & \quad f \quad g(f(x)) \\
\end{align*}
\]

\[
f \quad f(x) = \sqrt{x} \quad , \quad g(x) = \sqrt{2-x} \quad \text{find} \quad g \circ f, \quad f \circ g \quad \text{and domains.}
\]
<table>
<thead>
<tr>
<th>Function</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f )</td>
<td>([0, \infty))</td>
</tr>
<tr>
<td>( g )</td>
<td>((-\infty, 2])</td>
</tr>
<tr>
<td>( f \circ g )</td>
<td>((-\infty, 2])</td>
</tr>
<tr>
<td>( g \circ f )</td>
<td>([a, 4])</td>
</tr>
</tbody>
</table>

\[ \begin{align*}
\text{and} & \\
2 - \sqrt{x} & > 0 \quad x \geq 0 \\
4 & \geq x \quad x \geq 0
\end{align*} \]

\( f \circ g = \text{id} \) enityly

Show \( g \) is injective

\( f \) is surjective

\[ \frac{\sqrt{x}}{x} \]

\( f(g(x)) = f(g(\sqrt{x})) = f(a) \]

\[ \overset{\text{id}(x)}{=} \overset{\text{id}(\sqrt{x})}{=} \]

\[ x = \sqrt{x} \]

**Tangent**

Line \( l \) touches \( C \) at \( P \)

So line \( l \) is tangent to \( C \)
Algorithm to find tangent to a curve at a point

Example: \( y = x^2 \) (parabola)
\( (1, 1) \) (point)

Step 1: Find slope of \( \ell \)
Draw 'secants'

\[
\frac{y-1}{x-1} = \frac{x^2-1}{x-1}
\]

\[
\lim_{x \to 1} \frac{x^2-1}{x-1} = \lim_{x \to 1} x + 1 = 2
\]

Step 2: Find line, slope = 2
\( (1, 1) \) lies on it

\[
y - 1 = 2(x - 1) 
\]
Galileo's distance covered by a freely falling body is proportional to the square of time it has been falling.

\[ s(t) = 4.9t^2 \]

Find the velocity of a ball dropped from a tower of height 450m after 5 seconds.

\[
\lim_{t \to 0} \frac{s(5 + t) - s(5)}{t} = \frac{4.9(5^2) - 4.9(5)^2}{5}
\]

Difference quotient = important for derivatives

\[
= \frac{4.9(5^2 + 10t)}{t}
\]

\[
= \frac{4.9(t^2 + 10t)}{t}
\]

\[
= 4.9(t + 10)
\]

\[
\lim_{t \to 0} 4.9(t + 10) = 49
\]