Sample Quiz

N

5 Sep 2012

This test totals 0 points and you get 0 minutes to do it. If you are brave or don’t care, you can put down your name. Good luck!

1. Calculate the difference quotient of the functions
   (a) \( f(x) = x^3 \) at point \( x = a \)
   (b) \( f(x) = \frac{x+3}{x+1} \) at point \( x = 1 \)

2. Find the domain and the range of the following functions
   (a) \( f(x) = |2x + 3| \)
   (b) \( f(x) = e^x \)

3. If \( f = \sin(2x) \) and \( g = \frac{1}{x+3} \), find the following compositions and their domains
   (a) \( f \circ f \)
   (b) \( g \circ g \)
   (c) \( f \circ g \)

4. Find a quadratic equation with roots 1, 3 such that \( f(2) = 6 \)
Sample quiz solutions

1a) \( f(x) = x^3 \) at point \( x = a \)

\[
\text{Difference quotient} = \frac{(a+h)^3 - a^3}{h}
\]

\[
= \frac{a^3 + h^3 + 3a^2h + 3ah^2 - a^3}{h}
\]

\[
= \frac{h^3 + 3a^2h + 3ah}{h}
\]

\[
= \frac{h^2 + 3a^2 + 3ah}{h}
\]

1b) \( f(x) = \frac{x+3}{x+1} \) at point \( x = 1 \)

\[
\text{Difference quotient} = \frac{f(1+h) - f(1)}{h}
\]

\[
= \frac{\frac{1+h+3}{1+h+1} - \frac{1+3}{1+1}}{h}
\]

\[
= \frac{\frac{h+4}{h+2} - 2}{h}
\]

\[
= \frac{(h+4) - 2(h+2)}{h(h+2)} = -\frac{h}{h(h+2)}
\]
2) a) \( f(x) = |2x + 3| \)

Domain = \( \mathbb{R} \)
Range = \( [0, \infty) \)

Absolute value is always \( \geq 0 \)

Graph of \( |2x + 3| \) with points at \( x = 3 \) and \( x = -3/2 \).

b) \( f(x) = e^x \)

Domain = \( \mathbb{R} \)
Range = \( (0, \infty) \)

Exponential function never takes 0 or negative values.

Graph of \( e^x \).

3) a) \( f \circ f(x) = f(\sin 2x) \)

= \( \sin (2 \sin 2x) \)

Domain = \( \mathbb{R} \)
Range = \( [-1, 1] \)

b) \( g \circ g(x) = g\left(\frac{1}{x+3}\right) \)

= \( \frac{1}{\frac{1}{x+3} + 3} \)

= \( \frac{1}{\frac{1 + 3(x+3)}{x+3}} \)

= \( \frac{x+3}{3x+10} \)
Domain = \( \mathbb{R} \setminus \left\{ \frac{-10}{3} \right\} \)

because denominator \( 3x + 10 = 0 \) if \( x = -\frac{10}{3} \)

\[ \text{Range} = \mathbb{R} \setminus \left\{ \frac{1}{3} \right\} \]

[This was a bit hood, so if you don't get this, don't panic!]

\( \frac{x+3}{3x+10} = a^k \) will mean

\[ x+3 = 3ax + 10a, \quad \text{so} \quad x - 3ax = 10a - 3 \]

so \( x(-3a+1) = 10a - 3 \)

so \( x = \frac{10a - 3}{-3a + 1} \)

Denominator vanishes if \( -3a + 1 = 0 \), ie \( a = \frac{1}{3} \)

Check: if \( \frac{x+3}{3x+10} = \frac{1}{3} \), then \( 3(x+3) = 3x + 10 \)

then \( 3x + 9 = 3x + 10 \)

\( \Rightarrow \) \( 9 = 10 \) (not possible)
\[ f \circ g(x) = f \left( \frac{1}{x+3} \right) = \sin \left( \frac{2}{x+3} \right) \]

**Domain** = \( \mathbb{R} \setminus \{-3\} \)

as denominator \( x+3 = 0 \) if \( x = -3 \)

**Range** = \([-1, 1]\)

4) Guess that quadratic equation with roots 1, 3 must look like

\((x-1)(x-3)\)

Plug in \(a\), we get

\((2-1)(2-3) = -1\)

but we need 6 = -1 \( \times \) -6

so

\[-6 \times (x-1)(x-3) \text{ will work}\]