1 Differentiation

Multiplication rule : \((fg)' = f'g + g'f\)
Division rule : \((\frac{f}{g})' = \frac{f'g - g'f}{g^2}\)
Chain rule : \((f \circ g)'(x) = f'(g(x))g'(x)\)
Power rule : Derivative of \(x^n\) is \(nx^{n-1}\)
Extended power rule : Derivative of \(f(x)^n\) is \(n f(x)^{n-1} f'(x)\)
Slope : Slope of the tangent to \(y = f(x)\) at \((a, f(a))\) is \(f'(a)\).

2 Trignometric stuff

1. Derivative of sin is cos, derivative of cos is \(-\sin\).
2. Derivative of tan is \(\sec^2\). Derivative of cot is \(-cosec^2\)
3. \(\sin^2 x + \cos^2 x = 1\)
4. \(\sin(a + b) = \sin a \cos b + \cos a \sin b\)
5. \(\cos(a + b) = \cos a \cos b - \sin a \sin b\)
6. \(\sin(-a) = -\sin a\)
7. \(\cos(-a) = \cos a\)
8. \(\lim_{x \to 0} \frac{\sin x}{x} = 1\). Same for \(\frac{x}{\sin x}\).
3 Implicit differentiation, related rates

If \( y \) is a function of \( x \), \( \frac{d}{dx} y^2 = 2y \frac{dy}{dx} = 2yy' \) etc. For the related rates problem, draw a figure and give names to variables. Write down the given data in terms of these variables and write down what you have to find out. Implicitly differentiate and plug in values at the end to get your answer.

Remember Pythagoras theorem \( a^2 + b^2 = c^2 \), where \( a, b, c \) are the lengths of the sides of a right triangle and \( c \) is the longest side. Also remember that \( \frac{\sin x}{\cos x} = \tan x \) and \( \sin \) is the ratio of the opposite side to the hypotenuse, \( \cos \) is the ratio of the adjacent side to the hypotenuse in a right triangle.

4 Theorems

IVT If \( f \) is continuous on \([a, b]\), given any number \( N \) between \( f(a) \) and \( f(b) \), you can find an \( x \) in \([a, b]\) such that \( f(x) = N \).

Use: This helps to show many functions have roots. Find \( a \) such that \( f(a) < 0 \) and \( b \) such that \( f(b) > 0 \).

EVT If \( f \) is continuous on \([a, b]\), \( f \) has an absolute maxima and an absolute minima in \([a, b]\)

Use: This is why we have an algorithm to find absolute maxima and minima for nice functions

Rolle’s If \( f \) is continuous on \([a, b]\) and differentiable on \((a, b)\) and \( f(a) = f(b) \), then there is a \( c \) in \((a, b)\) such that \( f'(c) = 0 \). [Pictorially, if you start and end at the same level, at some point in between, the tangent is horizontal]

Use: This helps to show some functions have exactly one real root. The way to show that is to do it by contradiction. If there are 2 roots, \( a, b \) then \( f(a) = f(b) = 0 \), find \( f' \) and try to show it can never be 0.

MVT If \( f \) is continuous on \([a, b]\) and differentiable on \((a, b)\), then \( \frac{f(b)-f(a)}{b-a} = f'(c) \) for some \( c \) in \((a, b)\). [Pictorially, if you move the secant joining \((a, f(a))\) to \((b, f(b))\) parallelly, it will become a tangent to some point in between]

Use: If \( f' \) is less than (or greater than) some number, and you know \( f(a) \), you can give some estimate for \( f(b) \).
5 How to find the abs max,min

1. This procedure is for continuous $f$ on $[a, b]$ which by EVT have abs max,min in the domain

2. Find $f'$ and find out the points at which it is 0, and where it does not exist. These are called critical points

3. To this list of numbers, also add the end points of the domain, namely $a$ and $b$

4. Find $f$ values of each of these numbers and pick maximum and minimum among these values and their $x$-coordinates

6 Four tests

These two determine the shape of $f$:

(a) **Inc-dec test**: If $f' > 0$ on an interval, then $f$ is increasing there and if $f' < 0$ on an interval, then $f$ is decreasing there

(b) **Concavity test**: If $f'' > 0$ on an interval, then $f$ is concave up there and if $f'' < 0$ on an interval, then $f$ is concave down there

These two help you to find local maxima-minima:

(a) **First derivative test**: If $c$ is a critical point of continuous $f$, then if $f'$ changes sign from minus to plus at $c$, $f$ has a local minima at $c$. If $f'$ changes sign from plus to minus at $c$, then local maxima at $c$. If $f'$ doesn’t change sign, neither local maxima nor local minima at $c$.

(b) **Second derivative test**: If $f''$ is continuous near $c$ and $f'(c) = 0$, then if $f''(c) > 0$, there is a local minima at $c$. If $f''(c) < 0$, local maxima at $c$. If $f''(c) = 0$, the test is inconclusive. You should use the first derivative test at $c$ instead.
7 Practice Problems

Section 2.3 : (17,27,32,61,67,68,77,78)
Section 2.4 : (5,13,18,24,33,43,44,53(a))
Section 2.5 : (17,24,35,39,44,46,59,62,67)
Section 2.6 : (6,13,20,22,26,30,32,39)
Section 2.7 : (9,10)
Section 2.8 : (5,16,24,30,35,45)
Section 3.7 : (3,4,7,8,20,3,53,54)
Chapter 3 (Review) : Pages 276-277 Exercises (1-11,33,34,35,38,40,42)