1 Function domains

The basic rules are that you can’t divide by zero and that you can’t take square roots of negative numbers. Find the domains of the following functions

1. \( \sqrt{x - 3} \)
2. \( \sqrt{x^2 - 4} \)
3. \( \sqrt{x - 3} + \sqrt{x - 4} \)
4. \( \cot x \)
5. \( \frac{1}{\sin x + 2} \)
6. \( \frac{x - 1}{(x^2 + x + 12)(x^2 - 8)} \). Also find this function’s vertical asymptotes.
7. \( \frac{3}{y^4 + 7} \). Does this have any vertical asymptotes?

2 Function compositions

Find \( f \circ g \) and \( g \circ f \) for the following pairs of functions \( f, g \). Also find \( \lim_{x \to 0} f \circ g \) and \( \lim_{x \to 0} g \circ f \)

1. \( f(x) = \cos x, \ g(x) = \frac{1}{x} \)
2. \( f(x) = x - 2, \ g(x) = \frac{1}{x+1} \)
3 Equation of lines

1. Find the line passing through \((3, 4)\) with slope 2. Find another point on the line.

2. Find the line passing through \((1, 2)\) and \((-3, 1)\).

3. Find the slope of the line passing through \((x, y)\) and \((2, -1)\).

4 Reading graphs

Look at the following figure and answer the following questions.

1. Write down the function values at the points \(x = (-4), (-1), 2, 4\)

2. Write down the left limits of \(f\) at \(x = (-4), (-1), 2, 4\)

3. Write down the right limits of \(f\) at \(x = (-4), (-1), 2, 4\)

4. Write down the limits of \(f\) at \(x = (-4), (-1), 2, 4\)

5. Write down the 2 conditions for \(f\) to be continuous at \(a\)

6. Write down where our given function \(f\) is discontinuous and explain which condition it fails to satisfy.
5 Limits

Find the following easy limits

1. \( \lim_{x \to 1} \frac{1}{x-1} \)
2. \( \lim_{x \to 1^+} \frac{1}{x-1} \)
3. \( \lim_{x \to 1} \frac{1}{x-1} \)
4. \( \lim_{x \to 0} \frac{1}{\sin x} \)
5. \( \lim_{x \to 0} \sin \left( \frac{1}{x} \right) \)
6. \( \lim_{x \to 0} \frac{x}{\sin x} \)
7. \( \lim_{x \to 0} \frac{x}{\cos x} \)

Find the limits of the following. Write down all your steps clearly

1. \( \lim_{t \to 15} \frac{t-15}{\sqrt{t}-\sqrt{15}} \) [Hint: \( x^2 - a^2 = (x - a)(x + a) \)]
2. \( \lim_{x \to 3} \frac{x-3}{3x^2-8x-3} \) [Hint: Factorize and cancel. You must know how to factorize easy polynomials!]
3. \( \lim_{x \to 0} \left( x^2 \sin \left( \frac{1}{x} \right) \right) \) [Hint: Think Sandwiches]
4. \( \lim_{x \to -3} \frac{3-|x|}{3+|x|} \)
5. \( \lim_{x \to -3} \frac{3+|x|}{3+x} \) [You have to know when \( |x| = x \) and when \( |x| = (-x) \), and be careful with right and left limits]
6 More on continuity and Intermediate Value Theorem

Find the value of $a, b$ so that the following function is continuous

$$f(x) = \begin{cases} 
ax^2 + bx & \text{if } x \geq 2 \\
-ax & \text{if } 1 < x < 2 \\
-bx^2 - 4 & \text{if } x \leq 1 
\end{cases}$$

You must know the statement of IVT and must also know how to use it in problems. Show that there is a solution to the following equations in the given intervals

1. $x^2 - x = \sin x$ in $(-1, 1)$
2. $x^3 + 27 = 0$ in $(-4, -2)$

7 Derivatives using difference quotients

You must know that the difference quotient of $f$ at $x$ is $\frac{f(x+h) - f(x)}{h}$ and that the derivative of $f$ at $x$ is $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. Also you must know that the slope of the tangent to $y = f(x)$ at the point $(x, f(x))$ is $f'(x)$. Now compute the derivatives of the following functions at the given points using the difference quotient method. And using these answers, find the equation of the tangents to the functions at the given points.

1. $f(x) = x^3 + 2$ at $x = 1$
2. $f(x) = \frac{-3x}{\sqrt{x^2+2}}$ at $x = 5$
3. $f(x) = \sqrt{x^2 + 3}$ at $x = 0$
8 Derivatives using differentiation rules

You must know the all differentiation rules by-heart. Using them find \( h'(x) \) for the following functions

1. \( h(x) = \sqrt{x^7} \)
2. \( h(x) = \frac{x^2+3x+2}{x-1} \)
3. \( h(x) = \frac{x^2+3x+2}{x+1} \)
4. \( h(x) = \frac{1}{\sqrt{x^2+7}} \) [Use the division rule!]

5. You are given that the derivative for \( f(x) = \sin x \) is \( f'(x) = \cos x \). You are also given that the derivative of \( g(x) = \cos x \) is \( g'(x) = -\sin x \). Using the information given, find the following derivatives using multiplication or division rules as appropriate.

   (a) \( h(x) = \sin x \cos x \)
   (b) \( h(x) = \frac{x \sin x}{3x^2+1} \)
   (c) \( h(x) = \tan x \) [Hint : \( \tan = \frac{\sin}{\cos} \)]
   (d) \( h(x) = \cot x \) [Hint : \( \cot = \frac{\cos}{\sin} \)]
   (e) \( h(x) = \frac{\tan x}{x^2+1} \)