Quiz 10

7 Dec 2012

This test totals 25 points and you get 30 minutes to do it. Good luck!

1. (10 pts) Find the following integrals

(a) \( \int_{1}^{3} \frac{3}{(t-4)^2} \, dt \)

\[ u = t - 4 \]
\[ du = dt \]

\[ \int_{-3}^{-1} \frac{du}{u^2} = \frac{u^{-1}}{-1} \bigg|_{-3}^{-1} = \frac{1}{u} \bigg|_{-3}^{-1} = \frac{1}{1} - \frac{1}{-3} = 1 - \frac{1}{3} = \frac{2}{3} \]

(b) \( \int \sin(\pi t) \cos(\pi t) \, dt \)

\[ u = \sin(\pi t) \]
\[ du = \pi \cos(\pi t) \, dt \]

\[ \int \frac{u \, du}{\pi} = \frac{u^2}{2\pi} + c \]

\[ = \frac{(\sin(\pi t))^2}{2\pi} + c \]
2. (5 pts) Find the area between $y = 2x$ and $y = x^2 - 4x$ (Hint: $y = 2x$ lies above the other curve in the region of interest)

Intersection points

$y = 2x \quad y = x^2 - 4x$

$2x = x^2 - 4x \quad 6x = x^2 \quad x = 0 \text{ or } 6$

$y = 0 \text{ or } 12$

\[
\int_0^6 2x - (x^2 - 4x) \, dx = \int_0^6 6x - x^2 \, dx
\]

\[
= 6 \left( \frac{2x^2}{2} \right) - \frac{x^3}{3} \bigg|_0^6
\]

\[
= 6 \cdot 36 - \frac{6^3}{3}
\]

\[
= 6^2 \left( \frac{1}{2} - \frac{1}{3} \right)
\]

\[
= 6^2 \left( \frac{1}{6} \right) = 6^2 = 36 \text{ square units}
\]
3. (5 pts) Find the area bounded by $x = y^2 - 1$, $x = \sqrt{y}$, $y = 0$ and $y = 1$.
(Hint: $x = y^2 - 1$ lies to the left of $x = \sqrt{y}$ in the region of interest)

\[\int_{0}^{1} x_r \, dy = \int_{y=0}^{1} \sqrt{y} - (y^2 - 1) \, dy\]

\[= \int_{0}^{1} \sqrt{y} + 1 - y^2 \, dy\]

\[= 2y^{3/2} + y - \frac{y^3}{3} \bigg|_{0}^{1}\]

\[= \frac{2}{3} + 1 - \frac{1}{3} = 1 + \frac{1}{3} = \frac{4}{3} \text{ sq units}\]
4. (5 pts) Using integration, find the volume of the solid obtained by rotating the region under \( y = 3x \) from 0 to 1, about the x-axis.

\[
\int_0^1 \pi y^2 \, dx = \int_0^1 \pi (3x)^2 \, dx = 9\pi x^2 \bigg|_0^1 = 9\pi - \frac{9\pi}{3} = \frac{9\pi}{3} = 3\pi
\]