Quiz: The Fifth

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12 Oct 2012

This test totals 25 points and you get 25 minutes to do it. Good luck!

1. (10 pts) Find $\frac{dy}{dx}$ by implicit differentiation

(a) $y^2 + \cos(x + y) = 0$

$$2y \frac{dy}{dx} - \sin(x + y) \left( 1 + \frac{dy}{dx} \right) = 0$$

$$2y \frac{dy}{dx} - \sin(x + y) - \sin(x + y) \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} \left( 2y - \sin(x + y) \right) = \sin(x + y)$$

$$\frac{dy}{dx} = \frac{\sin(x + y)}{2y - \sin(x + y)}$$
(b) \( \cos x \sin y + \sin x \cos y = y + x \). Simplify as much as possible! The final answer is a number.

\[
\sin(y + x) = y + x
\]

\[\text{Implicit diff:} \quad \cos(y + x) \left( \frac{dy}{dx} + 1 \right) = \left( \frac{dy}{dx} + 1 \right)\]

\[\Rightarrow \quad \cos(y + x) \frac{dy}{dx} + \cos(y + x) = \frac{dy}{dx} + 1\]

\[\Rightarrow \quad \frac{dy}{dx} \left( \cos(y + x) - 1 \right) = 1 - \cos(y + x)\]

\[\Rightarrow \quad \frac{dy}{dx} = \frac{1 - \cos(y + x)}{\cos(y + x) - 1} = -1\]

\[\text{[If } \cos(y + x) \neq 1\text{]}\]
3. (3 pts) Find the tangent to \( \sin x + \cos x = y \) at the point \((0, 1)\).

\[
\cos x - \sin x = \frac{dy}{dx}
\]

\[
\left. \frac{dy}{dx} \right|_{x=0} = \cos 0 - \sin 0
\]

\[= 1 - 0 = 1 = m\]

\[y = mx + c\]

\[y = x + c\]

\[1 = 0 + c\]

\[\therefore c = 1\]
2. (10 pts) A ball is thrown up vertically from the ground. Its height function is given by \( h(t) = 20t - 5t^2 \). Using this information, answer the following questions

(a) Determine the ball’s velocity function.

\[ v(t) = 20 - 10t \]

(b) What is the maximum height attained?

\[ 20 - 10t = 0 \quad \Rightarrow \quad t = 2 \]

\[ h(2) = 20(2) - 5(2)^2 = 20 \]

(c) What is the initial velocity of the ball (i.e., with what velocity is it thrown up in the air?)

\[ v(0) = 20 - 10(0) = 20 \]

(d) What is the final velocity of the ball (i.e., with what velocity does it hit the ground again?)

\[ h = 0 \quad \Rightarrow \quad 20t = 5t^2 \quad \Rightarrow \quad t = 4 \]

\[ v(4) = 20 - 10(4) = -20 \]

(e) At \( t = 1.5 \) seconds, is it travelling up or down? What about at \( t = 3 \) seconds? Explain your answer.

\[ t < 2 \quad \uparrow \quad \quad t = 2 \quad 2 < t < 4 \quad \downarrow \quad \quad t = 1.5 \quad \uparrow \]

\[ \text{At } t = 1.5, \text{ the velocity is negative, so it is travelling down.} \]

\[ \text{At } t = 3, \text{ the height is } h(t) = 20t - 5t^2 = 0 \text{, so it is at the ground.} \]
4. (2 pts) Using information from the following figure (graph of a function $f$), answer the following questions

(a) $f'(-1) = \underline{0}$  \hspace{1cm} (Give an actual number as your answer!)
(b) Is $f'(0)$ positive or negative? (Explain your answer)  \hspace{1cm} \text{negative}

\text{At } 0, \quad \text{tangent}

\text{slope } < 0

\text{so } \quad f'(0) < 0