Quiz : The Fifth

N

12 Oct 2012

This test totals 25 points and you get 25 minutes to do it. Good luck!

1. (10 pts) Find \( \frac{dy}{dx} \) by implicit differentiation

(a) \( y^2 + \sin(x + y) = 0 \)

\[
2y \frac{dy}{dx} + \cos(x+y) \left( 1 + \frac{dy}{dx} \right) = 0
\]

\[
2y \frac{dy}{dx} + \cos(x+y) + \cos(x+y) \frac{dy}{dx} = 0
\]

\[
\frac{dy}{dx} \left( 2y + \cos(x+y) \right) = -\cos(x+y)
\]

\[
\frac{dy}{dx} = \frac{ - \cos(x+y) }{ 2y + \cos(x+y) }
\]
(b) \( \cos x \sin y - \sin x \cos y = y - x \). Simplify as much as possible! The final answer is a number.

\[
\cos x \sin y - \sin x \cos y = y - x
\]

\[
\Rightarrow \sin (y - x) = y - x
\]

\[
\text{Implicit diff}
\]

\[
\cos (y - x) \left( \frac{dy}{dx} - 1 \right) = \frac{dy}{dx} - 1
\]

\[
\cos (y - x) \frac{dy}{dx} - \cos (y - x) = \frac{dy}{dx} - 1
\]

\[
\Rightarrow \frac{dy}{dx} \left( \cos (y - x) - 1 \right) = \cos (y - x) - 1
\]

\[
\Rightarrow \frac{dy}{dx} = \frac{\cos (y - x) - 1}{\cos (y - x) - 1} = 1
\]

(If \( \cos (y - x) \neq 1 \))

Answer

\[
\cos x \sin y - \sin x \cos y = y - x
\]

\[
(\sin x) \sin y + \cos x \cos y \frac{dy}{dx} = \frac{dy}{dx} - 1
\]

\[
- \left( (\cos x) \cos y - \sin x \sin y \frac{dy}{dx} \right) = \frac{dy}{dx} - 1
\]

\[
\frac{dy}{dx} \left( \cos x \cos y + \sin x \sin y \right) = \frac{dy}{dx} - 1
\]

\[
\Rightarrow \frac{dy}{dx} \left( \cos x \cos y + \sin x \sin y - 1 \right) = \cos x \cos y + \sin x \sin y - 1
\]

\[
\therefore \frac{dy}{dx} = 1.
\]
2. (10 pts) A ball is thrown up vertically from the ground. Its height function is given by \( h(t) = 40t - 10t^2 \). Using this information, answer the following questions.

(a) Determine the ball’s velocity function.

\[ v(t) = 40 - 20t \]

(b) What is the maximum height attained?

\[ 40 - 20t = 0, \quad t = 2 \]

\[ h(2) = 40(2) - 10(2)^2 = 80 - 40 = 40 \]

(c) What is the initial velocity of the ball (i.e., with what velocity is it thrown up in the air?)

\[ v(0) = 40 - 20(0) = 40 \]

(d) What is the final velocity of the ball (i.e., with what velocity does it hit the ground again?)

\[ 40t - 20t^2 = 0, \quad 4 = t \]

\[ v(4) = 40 - 20(4) = -40 \]

(e) At \( t = 1.5 \) seconds, is it travelling up or down? What about at \( t = 3 \) seconds? Explain your answer.

\[
\begin{array}{c}
\text{\( t < 2 \)} \quad \text{travels up} \\
\text{\( t = 2 \)} \quad \text{\( t = 1.5 \) sec \uparrow} \\
\text{\( t > 2 \)} \quad \text{travels down} \\
\text{\( t = 3 \) sec \downarrow}
\end{array}
\]
3. (3 pts) Find the tangent to \( \cos x - \sin x = y \) at the point \((0,1)\).

\[
- \sin x - \cos x = \frac{dy}{dx}
\]

\[
\frac{dy}{dx} \bigg|_{x=0} = -\sin 0 - \cos 0 = -1 = m
\]

\[
y = mx + c
\]

\[
y = -x + c
\]

\[
x = 0, \quad y = +1
\]

\[
+1 = c
\]

\[
y = -x + 1
\]
4. (2 pts) Using information from the following figure (graph of a function $f$), answer the following questions

(a) $f'(-1) = 0$  
   (Give an actual number as your answer!)

(b) Is $f'(-2)$ positive or negative? (Explain your answer)

- The tangent looks like
- Slope > 0
- So $f'(-2) > 0$