1. Tangent to

\[ x^2 + y^2 = 1 \] at \((0, 1)\)

\[ y = \sqrt{1-x^2} \]

Pictorially, tangent looks like \( y = 1 \)!

Mathematically way to do it

1. Tangent \( l \) passes through \((0, 1)\)
2. Want to find slope of \( l \)
3. Instead find slope of neighbouring line \( P \tilde{Q} \)

\[
\text{slope of } P \tilde{Q} = \frac{y-1}{x-0} = \frac{\sqrt{1-x^2} - 1}{x}
\]

4. \( Q \rightarrow P \) \hspace{1cm} \lim_{x \to 0} \frac{\sqrt{1-x^2} - 1}{x} \; \text{ doesn't exist!}

But take:

\[ \lim_{x \to 0} \frac{1-x^2}{x} \]

\[ \lim_{x \to 0} \frac{\sqrt{1-x^2} - 1}{x} \; \text{ isn't 0} \]
\[ y = mx + c \]

\[ m = 0 \]

\[ y = c \]

\(C_0, D\) lies on \( y = f \), plug in \( x = 0, y = 1 \) to get \( c = 1 \)

\[
\begin{array}{c}
8 = 1
\end{array}
\]
3) a) \( f(x) = x^3 \) at point \( x = a \)

\[
\text{Difference quotient} = \frac{(a+h)^3 - a^3}{h}
\]

\[
= \frac{a^3 + h^3 + 3a^2h + 3ah^2 - a^3}{h}
\]

\[
= \frac{h(h^2 + 3a^2 + 3ah)}{h}
\]

\[
= h^2 + 3a^2 + 3ah
\]

b) \( f(x) = \frac{x+3}{x+1} \) at point \( x = 1 \)

\[
\text{Difference quotient} = \frac{f(1+h) - f(1)}{h}
\]

\[
= \frac{1+h+3}{1+h+1} - \frac{1+3}{1+1}
\]

\[
= \frac{h+4}{h+2} - 2 = \frac{-1}{h+2}
\]

\[
= \frac{(h+4) - 2(h+2)}{h(h+2)} = \frac{-h}{h(h+2)}
\]
3) a) \( f(x) = |2x + 3| \)

Domain = \( \mathbb{R} \)

Range = \( [0, \infty) \)

absolute value is always \( \geq 0 \)

b) \( f(x) = e^x \)

Domain = \( \mathbb{R} \)

Range = \( (0, \infty) \)

exponential function never takes 0 or -ve values

4) a) \( f \circ f(x) = f(\sin 2x) \)

\[ = \sin (a \sin 2x) \]

Domain = \( \mathbb{R} \)

Range = \( [-1, 1] \)

b) \( g \circ g(x) = g\left(\frac{1}{x+3}\right) \)

\[ = \frac{1}{1 + 3(x+3)} = \frac{x+3}{3x+10} \]
Domain = \( \mathbb{R} \setminus \{-10/3\} \)

because denominator \( 3x+10 = 0 \) if \( x = -10/3 \)

Range = \( \mathbb{R} \setminus \{1/3\} \)

[This was a bit hard, so if you don't get this, don't panic!]

Reason: \( \frac{x+3}{3x+10} = \sqrt{a} \) will mean

\[ x+3 = 3ax + 10a \] so \( x - 3ax = 10a - 3 \)

So \( x(-3a+1) = 10a - 3 \)

So \( x = \frac{10a - 3}{-3a + 1} \)

Denominator vanishes if \(-3a+1=0\), ie \( a = \frac{1}{3} \)

Check: if \( \frac{x+3}{3x+10} = \frac{1}{3} \), then \( 3(x+3) = 3x + 10 \)

then \( 3x + 9 = 3x + 10 \)

\( \Rightarrow \) \( 9 = 10 \) (not possible)
5 \quad f \circ g(x) = f \left( \frac{1}{x+3} \right) \\
= \sin \left( \frac{2}{x+3} \right)

\text{Domain} = \mathbb{R} \setminus \{-3\}

\text{as denominator } x+3 = 0 \text{ if } x = -3

\text{Range} = [-1, 1]

Question: Give quadratic with roots 1, 3 and \( f(2) = 6 \).

4) Guess that quadratic equation with root 1, 3 must look like

\[(x-1)(x-3)\]

Plug in 2, we get

\[2-1 \quad 2-3 = -1\]

but we need 6 = \(-1 \times -6\)

So,

\[-6(x-1)(x-3) \text{ will work}\]