Quiz 1

14 Sep 2012

This test totals 25 points and you get 20 minutes to do it. Good luck!

1. (5 pts) Match the following by drawing clear arrows. Each correct pairing gets you a point. (Easy!)

\[
\begin{array}{|c|c|}
\hline
\text{lim}_{x \to 0} \frac{1}{x} & \infty \\
\text{lim}_{x \to 0} \frac{1}{x^2} & 20 \\
\text{lim}_{x \to 5} (x^2 - 5) & 1 \\
\text{lim}_{x \to 0} (|x| + 2) & 2 \\
\text{lim}_{x \to 0} \frac{\sin x}{x} & \text{Does not exist} \\
\hline
\end{array}
\]

(Two bonus points) What is \( \lim_{x \to 0} \frac{x^2}{\sin x} \)? Give reason!

\[
\lim_{x \to 0} \left( \frac{x}{\sin x} \right) = \lim_{x \to 0} \left( \frac{x}{x} \right) = 1 \times 0 = 0
\]
2. (10 pts) Match the following functions with their domains by drawing clear arrows. Each correct pairing gets two points.

<table>
<thead>
<tr>
<th>Function</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{\sin x + 2}$</td>
<td>$\mathbb{R} \setminus {-1}$</td>
</tr>
<tr>
<td>$\frac{1}{(2x-1)(3x-1)}$</td>
<td>$(-\infty, -4] \cup [4, \infty)$</td>
</tr>
<tr>
<td>$\cot x$</td>
<td>$\mathbb{R}$</td>
</tr>
<tr>
<td>$\sqrt{x^2 - 16}$</td>
<td>$\mathbb{R} \setminus {\pm \frac{1}{2}, \frac{3}{2}}$</td>
</tr>
<tr>
<td>$\frac{2}{x^2 + 2x + 1}$</td>
<td>All except multiples of $\pi$</td>
</tr>
</tbody>
</table>

3. (a) (2 pts) What is $\lim_{x \to -1} (x^2 + 3x + 2)$?

(b) (2 pts) Solve the quadratic $x^2 + 3x + 2 = 0$ and write down the roots if they exist.

$$(x + 2)(x + 1) = 0$$

$-2$ are roots

(c) (1 pt) Solve $y^4 + 3y^2 + 2 = 0$ and write down the roots if they exist.

No roots! All terms are even, can't get 0.

(d) (3 pts) Find the difference quotient $\frac{f(x+h) - f(x)}{h}$ of $f(x) = \frac{1}{x+1}$ at the point $x = 0$.

$$\frac{1}{h+1} - \frac{1}{h} = \frac{1 - \frac{1}{h+1}}{(h+1)} = \frac{-1}{(h+1)^2}$$

(e) (2 pts) Find the limit as $h \to 0$ of your answer above.

$$\lim_{h \to 0} \frac{-1}{h+1} = -1$$