Quiz 6

N

26 Oct 2012

This test totals 20 points and you get 25 minutes to do it. Good luck!

1. (2 pts) If $f$ has a local maxima or minima at $c$, then either

$$f'(c)$$ is \_0\_ or \_does not exist\_

2. (2 pts) **Bonus**: Does $f(x) = x^{2013} + x^{2011} + x$ have a local maxima or a local minima anywhere on the real line. Why or why not?

$$f'(x) = 2013 \cdot x^{2012} + 2011 \cdot x^{2010} + 1$$

$$\geq 1$$

So never 0! and always exist.

So no local max/ minima
3. (10 pts) Find absolute maxima and minima for \( f(x) = \sqrt{x}(3-x) \) on 
\([0,4]\). Show ALL your steps.

By EVT, there is some abs max/min

\[
f'(x) = \frac{1}{2\sqrt{x}} (3-x) + \sqrt{x}(-1)
\]

\[
= \frac{3-x}{2\sqrt{x}} - \sqrt{x}
\]

\[
= \frac{3-x - 2\sqrt{x}\sqrt{x}}{2\sqrt{x}}
\]

\[
= \frac{3-3x}{2\sqrt{x}}
\]

\[f'(x) = 0 \quad \text{if} \quad 3-3x = 0, \quad \text{i.e.} \quad x = 1
\]

\[f^{(2)}(x) \quad \text{DNE} \quad \text{if} \quad x = 0 \quad \text{negative number not in domain}
\]

Critical pts: 0, 1,

end pts: 0, 4

Pt x = 0: 0, 1, 4

\[
f(x) = 0 \quad \begin{array}{c|c|c}
1 & 1 & 4 \\
0 & 2 & 2 \\
\end{array}
\]

abs max: abs min

\[f\] has abs min at 4

abs max at 1
4. (a) (3 pts) State the Mean Value Theorem (also called Lagrange's theorem).

If $f$ is continuous on $[a, b]$ and differentiable on $(a, b)$, then there is a $c$ in $(a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

(b) (5 pts) If $f$ is continuous on $[0, 4]$, $f(0) = 1$ and $2 \leq f'(x) \leq 5$ for all $x \in (0, 4)$, then show that $9 \leq f(4) \leq 21$.

By MVT

for some $c \in (0, 4)$

$$f'(c) = \frac{f(4) - f(0)}{4 - 0} = \frac{f(4) - 1}{4}$$

As $2 \leq f'(c) \leq 5$

$$2 \leq \frac{f(4) - 1}{4} \leq 5$$

Adding 1

$$8 \leq f(4) - 1 \leq 20$$

$$\Rightarrow 9 \leq f(4) \leq 21$$