This test totals 18 points. Good luck!

1. (6 pts) Match the differential equations and the graphs of their solutions by drawing clear arrows.

\[ y' = -x \]

\[ y' = 0 \]

\[ y' = y^2 + 1 \]
2. (12 pts) Answer the following questions based on the differential equation (DE) given below

\[ y' = 2 - \sin x \quad \text{(DE)} \]

(a) (1 pt) Find the slope of the arrow at the point (0, 1000) in the direction field of the DE

\[ y'(0, 1000) = 2 - \sin 0 = 2 \]

Slope: \( \frac{2}{1000} \)

(b) (2 pts) Can you find a point where the arrow in the direction field of the DE is horizontal. If yes, give the coordinates of the point. If not, explain.

No!

Arrow horizontal \( \Rightarrow \quad y' = 0 \)

\[ \Rightarrow \quad 2 - \sin x = 0 \]

\[ \Rightarrow \quad \sin x = 2 \]

Can never happen for any \( x \).

(c) (2 pts) Give the coordinates of a point \((x_0, y_0)\) where the arrow is the steepest possible in the direction field of the DE.

\[ -\sin x \text{ is max when } \sin x = \min, \quad \text{use } -1 \]

\[ \Rightarrow \quad 2 - (-1) = 3 \]

\[ (x_0, y_0) = \left(-\frac{\pi}{2}, 0\right) \]

(d) (2 pts) If \( y = f(x) \) is a solution to the DE, circle ONE of the four options below which best describes \( f(x) \).

i. \( f \) is decreasing on \((-\infty, \infty)\)

\[ \text{ii. } f \text{ is increasing on } (-\infty, \infty) \quad y' > 0 \]

iii. \( f \) is a constant function

iv. \( f \) is decreasing on \((-\infty, 0)\) and increasing on \((0, \infty)\)
(e) (2 pts) Use Euler's method using a step size $h = 1$ to find an approximate value of $y(1)$ for the initial value problem

$$y' = 2 - \sin x, \quad y(0) = 0$$

$$y' = f(x, y) = 2 - \sin x$$

$$x_0 = 0 \quad y_0 = 0 \quad h = 1$$

$$y_1 = y_0 + (h) \cdot f(x_0, y_0)$$

$$= 0 + (1) \cdot (2 - \sin 0)$$

$$= 0 + 2$$

$$= 2$$

So

$$x_1 = x_0 + h = 0 + 1 = 1$$

$$y_1 = 2$$

$y(1) \approx 2$
(f) (3 pts) Finally, solve the initial value problem
\[ y' = 2 - \sin x, \quad y(0) = 0 \]

\[ y = \int (2 - \sin x) \, dx \]

\[ = 2x + \cos x + C \]

\[ y(0) = 0 \]

\[ \Rightarrow x = 0 \quad \text{gives} \quad y = 0 \]

\[ \therefore 0 = \cos 0 + C \]

\[ 0 = 1 + C \]

\[ \therefore C = -1 \]

\[ y(x) = 2x + \cos x - 1 \]