Exam III - Practice Problems

N

11 April 2015

Geometric series and Test for divergence (11.2)

1. Find whether the following series are convergent. If yes, to what do they converge to. If not, justify.

(a) \[ \sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}} \]

(b) \[ \sum_{k=1}^{\infty} \frac{k^2}{k^2 - 1} \]

2. Convert 1.53424242424242424242..... into a rational number

Integral Test and estimates (11.3)

1. State the Integral test.

2. Find whether the following series are convergent. Justify.

(a) \[ \sum_{n=1}^{\infty} \frac{\ln n}{n} \]

(b) \[ \sum_{n=1}^{\infty} ne^{-n} \]
3. State the remainder estimate for the Integral test

4. Find an upper bound for $R_{10}$ for $\sum_{n=1}^{\infty} \frac{1}{n^3}$

**Comparison tests (11.4)**

1. State the Comparison and the Limit comparison tests.

2. Find whether the following series are convergent. Justify.
   
   (a) $\sum_{n=1}^{\infty} \frac{1 + \sin n}{10^n}$
   
   (b) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n + 1}$

**Alternating test (11.5)**

1. State the Alternating test.

2. Find whether the following series are convergent. Justify.
   
   (a) $\sum_{n=1}^{\infty} (-1)^n \left( \frac{n}{10^n} \right)$
   
   (b) $\sum_{n=1}^{\infty} (-1)^n \left( \frac{3n - 1}{2n + 1} \right)$

3. State the remainder estimate for the Alternating test

4. Find an upper bound for $R_5$ for $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$
Absolute convergence, ratio and root tests (11.6)

1. State the Ratio and Root tests

2. Find whether the following series are absolutely convergent. Justify.

   (a) \[ \sum_{n=1}^{\infty} \left( \frac{\cos n}{n^2} \right) \]

   (b) \[ \sum_{n=1}^{\infty} \left( \frac{n^n}{n!} \right) \]

   (c) \[ \sum_{n=1}^{\infty} \left( \frac{(2n + 3)^n}{(3n + 2)^n} \right) \]

Power series (11.8-11.9)

   (a) Find the power series representation around 0 for \( f(x) = \frac{1}{x+10} \).

   Find its radius of convergence

   (b) Find the power series for \( \int f(x) = \int \frac{1}{x+10} \)

   (c) Find the power series for \( f'(x) = \frac{d}{dx} \left( \frac{1}{x+10} \right) \)

Taylor series (11.10)

   (a) Find the Maclaurin series for \( f(x) = \ln(1+x) \). Find its radius of convergence

   (b) Find the Taylor series for \( f(x) = \cos x \) around \( x = \pi \).
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<td>$\sum \frac{n-1}{2n-1}$</td>
<td>Test for div</td>
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<td>$\sum \frac{\sqrt{n^3+1}}{3n^2+4n^2+2}$</td>
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<td>$\sum ne^{-n^2}$</td>
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<td>$\sum (-1)^n \frac{n^3}{n^4+1}$</td>
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<td>$\sum \frac{1}{2+3^n}$</td>
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