Basic topics covered

→ what is a limit?
→ how to read limits off a graph.
→ when does a limit not exist

Right limit ≠ left limit

\[ f(x) = \frac{x}{|x|}, \quad x \neq 0 \]

\[ f(x) = \sin \left( \frac{\pi}{x} \right), \quad x \neq 0 \]

In both cases: \( \lim_{x \to 0} f(x) \) Does NOT exist - DNE

Caution

\[ f(0) = 0 \]
\[ \lim_{x \to 0} f(x) \text{ exists and } = 1. \]

Limiting value does not depend on function value at that point.
\[
\lim_{x \to 4} \frac{x+3}{x-4} = ?
\]

Step 1: Try plugging in.

\[
\begin{array}{cccc}
    x & : & 4.1 & 4.01 & 4.001 & 4.0001 \\
    x+3 & : & 7.1 & 7.01 & 7.001 & 7.0001 \\
    x-4 & : & 0.1 & 0.01 & 0.001 & 0.0001 \\
    \frac{x+3}{x-4} & : & 71 & 701 & 7001 & 70001 \\
\end{array}
\]

\[
\text{Step 2: } x \to 4^+
\]

\[
\frac{x+3}{x-4} \text{ shoots up!}
\]

\[
\lim_{x \to 4^+} f(x) \text{ would be } +\infty \text{ if this made sense.}
\]

\[
\text{Step 3: } x \to 4^-
\]

\[
\begin{array}{cccc}
    x & : & 3.9 & 3.999 & 3.9999 \\
    x+3 & : & 6.9 & 6.999 & 6.9999 \\
    x-4 & : & -0.1 & -0.01 & -0.001 \\
    \frac{x+3}{x-4} & : & -69 & -699 & -6999 \\
\end{array}
\]

\[
\text{" } \lim_{x \to 4^-} f(x) = -\infty \text{"}
\]

In any case, \( \lim_{x \to 4} f(x) \text{ Does Not Exist.} \)
continuity

→ what is continuity?

Graphically

Test: can you draw the graph without lifting your pencil off the paper?

Yes

f is cont

No

f is discont

Mathematically

\[ f(a) \text{ exists} \]
\[ \lim_{x \to a} f(x) \text{ exists} \]
\[ \lim_{x \to a} f(x) = f(a) \]

implies \( f \) is continuous at \( a \).

Finding limits of continuous functions is easy, just plug in values and evaluate!