minimize $Q = x^2 + y^2$ where $x - y = 10$

$y = x - 10$

$Q(x) = x^2 + (x - 10)^2$

$= x^2 + x^2 + 100 - 20x$

$= 2x^2 + 100 - 20x$

$Q'(x) = 4x - 20$

$x = 5$ critical pt.

@ $x = 5$ has to be a local min

Quick question

abo maxima? DNE!

$x \uparrow, y \uparrow, x^2 + y^2 \uparrow$
A 24 inch piece of string is cut in 2 pieces. One is a square, the other forms a circle. How should string be cut so that sum areas is min? max?

\[ 0 \leq x \leq 24 \]

\[ x + 24 - x \]

\[ A(x) = \pi \left( \frac{x}{2\pi} \right)^2 + \left( \frac{6 - x}{4} \right)^2 \]

\[ = \frac{\pi x^2}{4\pi^2} + \frac{(6 - x)^2}{16} \]

\[ = \frac{x^2}{4\pi} + \frac{36 + 2\frac{x^2}{16}}{16} - 3x \]

\[ A'(x) = \frac{2x}{4\pi} + \frac{2x}{16} - 3 \]

\[ = \frac{x}{2\pi} + \frac{x}{8} - 3 \]

\[ = x \left( \frac{1}{2\pi} + \frac{1}{8} \right) - 3 \]

\[ = x \left( \frac{8 + 2\pi}{16} \right) - 3 \]
\[ \lambda = 0 \]

\[ x = \frac{3 \times 16 \pi}{8 + 2\pi} = \frac{48\pi}{8 + 2\pi} \approx 10.55 \]

\[ x = 24 \]

\[ A(0) = 24^2 = 576 \text{ sq in} \quad \text{\(\leq\) Abs max.} \]

\[ A(\frac{48\pi}{8 + 2\pi}) \approx \text{Abs min} \]

\[ B \quad \pi \left(\frac{24}{2\pi}\right)^2 = \frac{576}{4\pi} = 12.8 \]

\[ \approx 40.77 \text{ sq in} \]

\[ \text{Abs min @} \quad x = \frac{48\pi}{8 + 2\pi}, \quad \text{use 18.55 inches for circle, rest for square} \]

\[ \text{Abs max @} \quad x = 0, \quad \text{only make square} \]
Homing pigeons (Pg 233, ex 5)

Pigeon requires a different energy/mile to fly over water than over land.

At what angle $\theta$ should the pigeon fly to conserve energy?

Rel bet $x$ and $\theta$

\[
\tan \theta = \frac{3}{x}
\]

\[
x = 3 \cot \theta
\]

\[
sin \theta = \frac{3}{y}
\]

\[
y = 3 \cosec \theta
\]

Dist/land

\[
8 - x = 8 - 3 \cot \theta
\]

Dist/water

\[
y = 3 \cosec \theta
\]
\[ E(\theta) = 6 \csc \theta + 8 - 3 \cot \theta \]

minimize \( E(\theta) \)

constant \( 0 \leq \theta \leq \frac{\pi}{2} \)

\[ E'(\theta) = -6 \csc \theta \cot \theta + 3 \csc^2 \theta \]

\[ = -6 \frac{\cos \theta}{\sin^2 \theta} + \frac{3}{\sin^2 \theta} \]

\[ = -6 \cos \theta + 3 \]

exist everywhere in \((0, \frac{\pi}{2})\)

critical pt \(-6 \cos \theta + 3 = 0\), \(\cos \theta = \pm \frac{1}{2}\) \(\theta = \frac{\pi}{3} + 2n\pi, n \in \mathbb{Z} \)
\[ E' = \frac{-6(\cos \theta - \frac{1}{2})}{\sin^2 \theta} \]

Rough graph sketch

\[ \theta = \frac{\pi}{3} \]

\[ y = 3 \cosec(\theta) = 3 \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2} \text{ miles} \]

\[ 8 - x = 8 - 3 \cdot \frac{1}{\sqrt{3}} = 8 - \sqrt{3} \text{ miles on land.} \]