Exponential Growth model

Graph

Initial population

Diff Eq
\[ \frac{dP}{dt} = kP \quad k > 0 \]

\( k = \text{growth rate} \)

Solution
\[ P(t) = Ce^{kt} \]

\( P(0) = C = \text{initial population} \)

Doubling time
\[ T_{\text{doubling time}} = \frac{\ln 2}{k} \]

ex: \( P(t) = 20e^{0.7t} \)

A population grows exp at 70% growth rate. Initially its size is 20

Half life
\[ t_{\text{half life}} = \frac{\ln 2}{k} \]

Exponential decay model

Graph

Initial decay

Diff Eq
\[ \frac{dQ}{dt} = -kQ \quad k > 0 \]

ex: \( \frac{dQ}{dt} = -0.05Q \)

\( k = \text{decay rate} \)

Solution
\[ Q(t) = Ce^{-kt} \]

\( Q(0) = C = \text{initial decay} \)

Half life
\[ t_{\text{half life}} = \frac{\ln 2}{0.05} \]

A qy decays exp at rate 5%, Initially 20gms present
Newton's law of cooling

\[ T(t) = \text{temp of object at time } t \quad T(0) = \text{initial temp of object} \]

\[ C = \text{(constant) surrounding temp} \]

**Diff eq**

\[ \frac{dT}{dt} = -k(T - C) \quad k > 0 \]

**Solution**

\[ T(t) = P_0 e^{-kt} + C \]

\[ P_0 = T(0) - C = \text{initial temp} - \text{surrounding temp} \]

\[ C = \text{(constant) surrounding temp} \]

**Ex:** A glass of water at 100°C placed in a room at 25°C

\[ T(t) = 75 e^{-kt} + 25 \]

\[ \frac{dT}{dt} = -k(T - 25) \]