1. \[ y' = 4x - 5 \quad \text{and} \quad \gamma(-1) = 3 \]

\[ y = \int (4x-5) \, dx \]

\[ y = \frac{4x^2}{2} - 5x + C \]

\[ y = 2x^2 - 5x + C \]

**IVP:** Find \( C \)

\[ 3 = y(-1) = 2 + 5 + C \]

\[ C = -4 \]

**Answer:** \( y = 2x^2 - 5x - 4 \)

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2. \[ y' = \cos x \sqrt{4 - 3 \sin x} \quad \quad y(\pi/2) = -1 \]

\[ y' = \cos x \sqrt{4 - 3 \sin x} \]

\[ y = \int \cos x \sqrt{4 - 3 \sin x} \, dx \]

**SUB**

\[ u = 4 - 3 \sin x \]

\[ du = -3 \cos x \, dx \]

\[ \int \sqrt{u} \, \frac{du}{-3} = \int \frac{u^{1/2}}{-3} \, du = \frac{-1}{3} u^{3/2} \cdot \frac{3}{2} + C = \frac{-1}{2} (4 - 3 \sin x)^{3/2} + C \]

\[ y = \frac{-1}{2} (4 - 3 \sin x)^{3/2} + C \]

Solve for \( C \): \( y(\pi/2) = -1 = \frac{-1}{2} (1) + C \), \( C = -1/2 \)
3. \[ y' = 5(x - y) \quad \text{and} \quad y(0) = 2 \]

\text{LFODE Method 2}

* \[ y' = 5x - 5y \Rightarrow y' + 5y = 5x \]
  \[ y' + p(x)y = q(x) \]

* \[ p(x) = 5, \quad q(x) = 5x \]

* \[ F(x) = \int p(x)\,dx = \int 5\,dx = 5x \]

* \[ g(x) = e^{F(x)} = e^{5x} \]

* \[ \int g(x)q(x)\,dx = \int e^{5x} \cdot 5x\,dx \]

\textbf{PARTS:} \[ u = 5x, \quad du = 5\,dx \]
  \[ dv = e^{5x}, \quad v = \frac{e^{5x}}{5} \]

\[ \int u\,dv = uv - \int v\,du = (5x) \frac{e^{5x}}{5} - \int \frac{e^{5x}}{5} \cdot 5\,dx \]

\[ = xe^{5x} - \frac{e^{5x}}{5} + C \]

\text{divided by } g(x)

* \text{Final ans:} \quad y(x) = \frac{xe^{5x} - \frac{e^{5x}}{5} + C}{e^{5x}}

\[ y(x) = x - \frac{1}{5} + Ce^{-5x} \]

\text{solve for } C, \quad y(0) = 2

\[ = -\frac{1}{5} + C, \quad C = 2 + \frac{1}{5} = \frac{11}{5} \]

\[ y(x) = x - \frac{1}{5} + \frac{11}{5}e^{-5x} \]
4. \[ x^2 y' + 4xy = 8x \] \[ y(1) = 4 \]

**LFODE, method 2**

divide by \(x^2\), \[ y' + \frac{4}{x}y = \frac{8}{x} \]

* \( p(x) = \frac{4}{x} \) \( q(x) = \frac{8}{x} \),

* \( F(x) = \int p(x)dx \)
  \[ = \int \frac{4}{x}dx = 4 \ln |x| \]

* \( G(x) = e^{F(x)} = e^{4 \ln |x|} = x^4 \)

* \( \int G(x)q(x)dx = \int x^4 \cdot \frac{8}{x}dx \)
  \[ = \int 8x^3dx = \frac{8}{4}x^4 + C \]

final ans = \[ \frac{2x^4 + C}{x^4} \]

\[ y(x) = 2 + Cx^{-4} \]

solve for \(C\)

\[ y(1) = 4, \]
\[ y(1) = 2 + C \implies C = 2 \]

\[ y(x) = 2 + 2x^{-4} \]
5. \[ y' = \frac{x^3}{1+4y^7}, \quad y(2) = -1 \]

\[ y' (1+4y^7) = x^3 \]

\[ \frac{dy}{dx} (1+4y^7) = x^3 \]

\[ \int (1+4y^7) \, dy = \int x^3 \, dx \]

\[ y + \frac{4y^8}{8} = \frac{x^4}{4} + C \]

\[ y + \frac{y^8}{2} = \frac{x^4}{4} + C \]

\[ y(2) = -1, \quad \leftarrow \lambda = 2, \quad y = -1 \]

\[ -1 + \frac{1}{2} = \frac{16}{4} + C \]

\[ \therefore \quad -\frac{1}{2} = 4 + C \]

\[ C = -4 \frac{1}{2} = -\frac{9}{2} \]

\[ y + \frac{y^8}{2} = \frac{x^4}{4} - \frac{9}{2} \]

\[ \uparrow \]

\[ \text{implicitly defined} \]
6. \[ y' = \frac{3x^2}{y} \quad y(1) = -2 \]

\[ y' \cdot y = 3x^2 \]

\[ \frac{dy}{dx} \cdot y = 3x^2 \]

\[ \int y \, dy = \int 3x^2 \, dx \]

\[ \frac{y^2}{2} = \frac{3x^3}{3} + C \]

\[ \frac{y^2}{2} = x^3 + C \]

\[ y^2 = 2x^3 + 2C \]

\[ y = \sqrt{2x^3 + 2C} \]

\[ y(1) = -2 \]

\[ -2 = \sqrt{2 + 2C} \]

\[ 4 = 2 + 2C \]

\[ 2 = 2C \]

\[ C = 1 \]

\[ y = \sqrt{2x^3 + 2} \]