Let \( \lim_{x \to a} f(x) = L \)
\( \lim_{x \to a} g(x) = M \)

and let \( c \) be a constant.

Then

#1 \( \lim_{x \to a} cf(x) = cL \)

example: \( \lim_{x \to 0} \frac{2 \sin x}{x} \)

\[
= 2 \times \lim_{x \to 0} \frac{\sin x}{x} \\
= 2 \times 1 \\
= 2
\]

#2 a) \( \lim_{x \to a} f(x) + g(x) = L + M \)

example: \( \lim_{x \to 3} \frac{2}{x} + \frac{x^2 - 9}{x - 3} \)

\[
= \lim_{x \to 3} \frac{2}{x} + \lim_{x \to 3} \frac{x^2 - 9}{x - 3} \\
= \frac{2}{3} + \lim_{x \to 3} (x + 3) \\
= \frac{2}{3} + 6 \\
= 6 \frac{2}{3}
\]

#2 b) \( \lim_{x \to a} f(x) - g(x) = L - M \)
#3 \[ \lim_{x \to a} \frac{f(x) \cdot g(x)}{x^2 \cdot (x+2)} = L \cdot M \]

\[ \text{ex:} \quad \lim_{x \to 0} \frac{\sin^2 x \cdot \cos x}{x^2 \cdot (x+2)} \]

\[ = \lim_{x \to 0} \left( \frac{\sin x}{x} \right)^2 \lim_{x \to 0} \frac{\cos x}{x+2} \]

\[ = 1 \cdot \frac{1}{2} \quad \text{Again you have to apply rule 3 here} \]

#4 \[ \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{L}{M} \quad \text{provided} \quad M \neq 0 \]

\[ \text{ex:} \quad \lim_{x \to 3} \frac{2x+7}{x-4} \]

\[ = \lim_{x \to 3} \frac{2x+7}{x-4} \]

\[ = \frac{13}{-1} \]

\[ = -13 \]

Note! If the problem was \[ \lim_{x \to 3} \frac{2x+7}{x-3} \], we couldn't have applied #4! Otherwise we will end up with \[ \frac{13}{0} \] ! XXXX
5) If \( H(x) \) is a function cont at \( x = L \)

then

\[
\lim_{x \to a} H(f(x)) = H(L) = H(x)
\]

\[
\lim_{x \to 3} e^{x^2 + 1} = e^{64}
\]

[ because \( e^x \) is continuous ]

Here \( H(x) = e^x \)
\(-f(x) = x^2 + 1\)

**Quick tips**

\[
\frac{0}{\text{non zero #}} = 0
\]

\[
\text{non zero #} \quad \text{← "±∞"}
\]

**TABOO**!

\[
\frac{0}{0} \quad ±∞, \quad 0 \times ±∞, \quad ±∞ - ±∞, \quad etc
\]

If you get any of this, it means you go back to the problem (and use a different strategy: ex: algebraic factorization etc.) and get the limit
\[
\lim_{x \to 10} \frac{7}{(x-10)} \left[ \frac{17x + 1}{(x-10)^2} \right]
\]

→ If you plug in, you get num = "∞", but \(\frac{∞}{∞}\) is taboo.

→ So you simplify!

\[
\lim_{x \to 10} \frac{7}{17x + 1}
\]

And now when you plug in, you get \(\frac{7}{171} \times 0 = \boxed{0}\).

\[
\lim_{x \to 0^+} \frac{x + x^2}{x \sqrt{x}}
\]

→ So you simplify

\[
\lim_{x \to 0^+} \frac{x(1+x)}{x \sqrt{x}}
\]

= \lim_{x \to 0^+} \frac{1+x}{\sqrt{x}}, \quad \text{plug in gets} \quad \frac{0}{0} \quad \text{taboo}

\[
x \to 0^+, \quad 1 + x \to 1^+; \quad \text{so} \quad \frac{1+x}{\sqrt{x}} \to +\infty\]

\[
\sqrt{x} \to 0^+, \quad \text{so} \quad \lim \text{ DNE ("+∞")}
\]
The following questions should be answered **without any use of a calculator**. You may use your notes and text.

We want to create a model for the length of the day, that is, the number of hours from sunrise to sunset, in London, England. We know that the shortest day is 7.5 hours and the longest day is 16.5 hours. We make the following assumptions:

- time $t$ is measured in days, $t \in [0, 365)$, with $t = 0$ corresponding to 21 December, which we assume is the shortest day of the year (known as the winter solstice);
- the day length, as a function of $t$, is denoted by $L(t)$, and is a sinusoidal function.

**Questions, Part I**

(1) Find a sinusoidal function for $L(t)$ that fits the information above.

(2) Sketch a graph of $L(t)$ on the domain $[0, 2\pi)$. Label axes and all points of interest.

(3) What value of $t$ maximizes $L(t)$? What day(s) of the year does this correspond to, roughly?

**Questions, Part II**

(4) Find the (instantaneous) rate of change of day length with respect to time.

(5) Sketch a graph of $L'(t)$ on the domain $[0, 2\pi)$. Label axes and all points of interest.

(6) When is this rate of change 0? What day(s) of the year does this correspond to, roughly?

(7) When is this rate of change maximized? What day(s) of the year does this correspond to, roughly?

(8) Some plants put out leaves in the spring in response to the change in day length. At what time of year would it be easiest for plants to detect changes in day length?