Practice with derivatives

Recall

1) \[ \frac{df}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \]

2) \[ \frac{df}{dx} \bigg|_{x=a} = f'(a) \]

3) \[ \frac{d}{dx} x^n = nx^{n-1} \]

4) \[ \frac{d}{dx} c \cdot f(x) = c \cdot \frac{d}{dx} f(x) \]

5) \[ \frac{d}{dx} c = 0 \]

6) \[ \frac{d}{dx} (f + g)(x) = \frac{d}{dx} f(x) + \frac{d}{dx} g(x) \]

7) Sharp corners \(\not\rightarrow\) not differentiable there, but cont!

- Jumps / holes \(\not\rightarrow\) not cont, not differentiable there

- \(f'(a) \text{ DNE}\) \(\not\rightarrow\) not cont, not differentiable there also

8) \[ \frac{d}{dx} \sin x = \cos x \]

9) \[ \frac{d}{dx} \cos x = -\sin x \]

10) Slope of tangent at \((a, f(a)) = f'(a) \)
I. Find $f'(x)$ using the definitions! [i.e., limit & difference quotient]'

1) $f(x) = 2x^2 + 3x - 2$
2) $f(x) = \cos(2x)$

II. Find $f'(x)$ using the rules to manipulate derivatives

1) $f(x) = \sin x + \cos x$
2) $f(x) = \sqrt{x} - \frac{1}{\sqrt{x}}$
3) $f(x) = \frac{2}{x} - \frac{x}{2}$
4) $f(x) = -x^3 + 8x^2 + 11$
5) $f(x) = \frac{\sqrt{x} \sin x - 2x + 3}{\sqrt{x}}$

III. Conceptual questions

Find $x$ so that
1) $f'(a) = 0$
2) $f'(a) = 0$
3) $f$ is continuous at $a$
4) $f$ is not differentiable at $a$
A car travels in a straight road in a single direction, which has a speed limit of 40 miles/hr. Assuming the car driver is a responsible law abiding citizen, answer the following:

(a) What is the max possible distance she could have covered in 3 hrs? In this situation, sketch distance vs. time graph.

(b) Which of the following graphs CANNOT be a plot of her car journey?

(c) While recounting her car trip, she told you that she started at 5 PM, and her car tire burst forcing her to stop the car at 6 PM. It took her 1 hour to fix the car after which she started off at 20 miles/hr. (Distance vs. time, miles vs. hrs)

Is \( s(t) \) differentiable at \( t = 2 \)?

Sketch \( s(t) \) for \( t = 1 \) to \( 2 \)

A polynomial \( f(x) \) has degree 5. What is the degree of its derivative? Rob found a polynomial whose derivative was 3, can you guess his polynomial?

He also found a polynomial whose derivative was \( 2x^2 + 3 \). Can you guess this polynomial?
let us pretend we are Newton sitting on apple trees instead of underneath it. We drop apples from different heights and get the following data:

<table>
<thead>
<tr>
<th>Apple #</th>
<th>ht from which it was dropped (m)</th>
<th>final velocity (m/s)</th>
<th>time taken (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>45</td>
<td>30</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>80</td>
<td>40</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>500</td>
<td>50</td>
<td>10</td>
</tr>
</tbody>
</table>

We have figured out the formula for distance, velocity:

\[ s(t) = at^2 + bt + c \]
\[ v(t) = dt + e \]

irrespective of the mass of the apple but have to find the constants!

Find the formulae, and acceleration of these apples...

Voila, gravity!