Quiz: The Fourth

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26 Sep 2014

This test totals 15 points and you get 15 minutes to do it. Good luck!

1. Choose the right answer for each of the following. JUSTIFY your answer by analysing the derivative/double derivative/graph. (7.5 pts)

(a) \( f(x) = x^5 + x + 1 \) is \underline{always increasing} \[ f'(x) = 5x^4 + 1 > 0 \] (i) always increasing (ii) always decreasing (iii) neither

(b) \( g(x) = 3x^2 \) is \underline{concave up always} \[ g''(x) = 6 > 0 \] (i) concave down always (ii) concave up always (iii) neither

(c) \( h(x) = \sin x + \cos x \) has a critical point at \( x = \underline{\frac{\pi}{4}} \)
   (i) \( \frac{\pi}{2} \) (ii) \( \frac{\pi}{3} \) (iii) \( \frac{\pi}{4} \)

(d) \( j(x) = x^7 - 7x - 6 \) has a \underline{rel minima} \[ j'(x) = 7x^6 - 7 \] at \( x = 1 \)
   (i) relative maxima (ii) relative minima (iii) neither

(e) An absolute maxima for \( \tan(x) \) on \( (-\frac{\pi}{2}, \frac{\pi}{2}) \) is attained at \( x = \underline{\text{no abs maxima}} \)
   (i) 0 (ii) \( \frac{\pi}{4} \) (iii) There is no absolute maxima
2. This is a two part question about a function \( f(x) = 2x^3 - 15x^2 + 36x + 4 \). SHOW WORK!

(a) Find all the relative extrema of \( f \). Use whichever derivative test suits you best. The domain for this part of the question is all of \( \mathbb{R} \). (5 pts)

\[
f'(x) = 6x^2 - 30x + 36
\]
\[
= 6(x^2 - 5x + 6) = 6(x-2)(x-3)
\]

Critical pts \( x = 2, x = 3 \)

\[
f''(x) = 12x - 30
\]

\[
f''(2) = 24 - 30 < 0 \quad \text{rel max at } x = 2
\]

\[
f''(3) = 36 - 30 > 0 \quad \text{rel min at } x = 3
\]

(b) For this part of the question alone, your domain of interest for \( f \) is given to be the closed interval \([-1, 1]\). Find all the absolute maxima and minima in this situation. (2.5 pts)

[Graph showing critical points and intervals]

\[
f(-1) = -2 -15 - 36 + 4 = -49 \quad \text{abs min at } x = -1
\]

\[
f(1) = 2 - 15 + 36 + 4 = 27 \quad \text{abs max at } x = 1
\]