lab 9

Probability

\[ P : \text{Sample space} \rightarrow [0, 1] \]

\[ \star \quad S \quad \rightarrow \quad 1 \]

\[ \star \quad \begin{align*}
A & \quad \rightarrow \quad a \\
B & \quad \rightarrow \quad b
\end{align*} \]

and \( A \cap B = \emptyset \) \text{ m.e. events/ disjoint sets} \rightarrow a+b

\[ \star \quad \begin{align*}
A & \quad \rightarrow \quad a \\
A^c & \quad \rightarrow \quad 1-a
\end{align*} \]

\[ P(A) + P(B) = P(A \cup B) \quad \text{and} \quad A \cap B = \emptyset \]

\[ P(A) + P(A^c) = 1 \]

\text{multiplication rule}

\[ P(A \cap B) = P(A) \frac{P(B|A)}{P(A)} \]

\[ \uparrow \]

Prob of \( B \) given \( A \).

Prob of \( A \) and \( B \) happening together.
Independent or not?

A, B are independent \iff \quad P(B|A) = P(B)

\quad \text{the fact that}

\quad A \text{ happens}
\quad \text{doesn't affect}
\quad \text{the probability of}
\quad B \text{ happening}

P(B|A) = P(B) \iff P(B|A) \cdot P(A) = P(A) \cdot P(B)

\iff P(A \cap B) = P(B) \cdot P(A)

\quad \uparrow

\quad \text{another way to check A, B are independent}

Coin flipping!

A fair coin is flipped 5 times.

1. \quad P(\text{all heads}) = \frac{1}{2^5} = \frac{1}{32}

2. \quad P(\text{no heads}) = \frac{1}{2^5} \quad \text{also}

3. \quad P(\text{not every flip is a head}) = 1 - \frac{1}{32} = \frac{31}{32}
4. Are the events
   - all heads = A
   - not all heads = B
   \( A \cap B = \emptyset \)
   - mutually exclusive? Yes
   - are they independent? No!
   \( P(A \cap B) = 0 \)
   \( P(A) \cdot P(B) = \frac{1}{32 \times 32} \neq 0 \).

Can mutually exclusive events be independent? No!
If A occurs, you are saying B can never occur.
\( P(B \mid A) \) is in fact 0!

Or \( P(A \cap B) = 0 \! . \)

5. Can you think of 2 events which are independent?
   - First coin = H
   - Second coin = H
   \( P(A) = \frac{1}{2} \)
   \( P(B) = \frac{1}{2} \)
   \( P(A) \cdot P(B) = \frac{1}{4} \)
   \( P(A \cap B) = P(\text{H--H}) = \frac{2^3}{2^5} = \frac{1}{4} \! . \)
Quick question:

what is \( P(ANB) + P(ANB^c) + P(A^cN^B) + P(A^cN^B^c) \)?  

In a box, you have 5 red chips labelled 1, \ldots, 5 and 5 green chips labelled 1, \ldots, 5. You draw 2 chips from it, one after the other without replacement.

Sample space = 10 \times 9 = 90

\[ P(1st \# = 1) = \frac{2}{10} = \frac{1}{5} \]

\[ P(2nd \# = 1 | 1st \# = 1) = \frac{1}{9} \]

\[ P(2nd \# = 1 | 1st \# \neq 1) = \frac{2}{9} \]
Draw the tree!

\[ \frac{1}{9} \rightarrow 1 \]
\[ \frac{4}{5} \rightarrow \neq 1 \]
\[ \frac{2}{9} \rightarrow 1 \]
\[ \frac{7}{9} \rightarrow \neq 1 \]

1st #

2nd #

\[ P(1\text{st }\# = 1 \text{ and } 2\text{nd }\# = 1) = \frac{1}{9} \times \frac{1}{5} = \frac{1}{45} \]

\[ P(2\text{nd }\# \neq 1 \text{ and } 1\text{st }\# = D) = P(B^c|A) \cdot P(A) \]
\[ = \frac{8}{9} \times \frac{1}{5} = \frac{8}{45} \]

Are \( A \) and \( A^c \) independent? No! \( P(A \cap A^c) = 0 \)
\[ P(A) \cdot P(A^c) = \frac{4}{25} \]

Are \( A \) and \( B^c \) independent? \( P(A) = \frac{1}{5} \)
\[ P(B^c) = \frac{1}{5} \times \frac{8}{9} + \frac{4}{5} \times \frac{7}{9} = \frac{36}{45} = \frac{4}{5} \]
\[ P(A) \) \dfrac{4}{25} \]
\[ P(A \cap B^c) = \dfrac{1}{5} \times \dfrac{8}{9} = \dfrac{8}{45} \]

So NOT independent! 
( common sense,
- the drawing of the 1st # affects
- the drawing of 2nd #)

F = dominant trait - beaxy purple flowers
f = recessive trait - white flowers

A type Ff plant is crossed with a plant of unknown genotype. It was observed that exactly half of the offspring bore purple flowers. Find the genotype of the plant in question.

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Purple flowers 

so have to bear white flowers

so \( \text{ ff } \)