1. Differentiation laws
Differentiate the following functions:
(a) \( f(x) = \frac{1}{5 - 2^x} \)
(b) \( g(x) = x^2 + 2^x \)
Solution: \( g'(x) = 2x + 2^x \ln(2) \)
(c) \( h(x) = e^3 \)
Solution: This is a constant function, so the derivative is 0.
(d) \( s(t) = \frac{t^2 + b}{t + a} \)
(e) \( x^2 + x^4 \)
Solution: use the power rule \( 2x + 4x^3 \)
(f) \( \sqrt{x} \sin(x) \)
Solution: product rule
\( \frac{1}{2}x^{-\frac{1}{2}} \sin(x) + \sqrt{x} \cos(x) \)
(g) \( (2x^4 + \cos(2x + 1))^2 \)
Solution: Chain rule \( 200(2x^4 + \cos(2x + 1))^199(8x^3 - \sin(2x + 1))2 \)
(h) \( \ln(\ln(\ln(x))) \)
Solution: Chain Rule
\( \frac{d}{dx} (\ln(\ln(x))) = \frac{1}{\ln(\ln(x))} \frac{d}{dx} (\ln(x)) = \frac{1}{\ln(\ln(x))} \frac{1}{\ln(x)} \frac{1}{x} \)
(i) \( \sin(\cos(\sin^2(x) + 3x)) \)
(j) \( e^{2\cos(x)} \)
Solution: Chain Rule
\( \frac{d}{dx} (e^{2\cos(x)}) = e^{2\cos(x)} \frac{d}{dx} (2\cos(x)) = e^{2\cos(x)} 2(-\sin(x)) \)

2. Applying derivatives to moving particles: If a particle is traveling along a line and its position is given by \( f(t) = \cos(e^{2t+1}) \), find an expression for the particle’s velocity and acceleration at time \( t \).
Solution: For the velocity, just compute \( f'(t) \) (use the chain rule), and for the acceleration, compute \( f''(t) \).

3. Find \( \frac{dy}{dx} \) in terms of \( x \) using logarithmic differentiation:
(a) \( y = \sqrt[3]{\frac{x^2}{x^4 + 1}} \)
Solution:
\( \ln(y) = \frac{1}{3} \ln(x - 1) - \ln(x^4 + 1) \)
\[ \frac{1}{y} \frac{dy}{dx} = \frac{1}{2} x^{-1} - \frac{1}{x^4 + 1} (4x^3) \]

\[ \frac{dy}{dx} = \frac{\sqrt{x^2 - 1}}{x^4 + 1} \left( \frac{1}{2} x^{-1} - \frac{1}{x^4 + 1} (4x^3) \right) \]

(b) \( y = (\cos(x)) \sin(x) \)
(c) \( y = e^{x^3 + x^2 - 1} \).

4. Find the equation of the tangent line at (0, -2) for the curve \( y^2(y^2 - 4) = x^2(x^2 - 5) \).

5. Find \( \frac{dy}{dx} \) by implicit differentiation: \( x \sin(y) + y \sin(x) = 1 \).

6. Indeterminate limits. Compute the following:
   (a) \( \lim_{x \to 0} \frac{\sin(3x)}{x} \)
   (b) \( \lim_{x \to 1} \frac{\sin(x - 1)}{x^2 + x - 2} \)
   (c) \( \lim_{x \to 0} \frac{\sin(x^2)}{x} \)
   (d) \( \lim_{x \to 0} \frac{\sin(5x)}{\sin(2x)} \)
   (e) \( \lim_{n \to \infty} (1 + \frac{1}{2})^{2n} \)
   (f) \( \lim_{n \to \infty} (1 + \frac{2}{n})^n \)

5. Find the absolute maximum and absolute minimum of \( 3x^4 - 4x^3 - 12x^2 + 1 \) on \([-2, 3]\).

6. Find the critical points of the functions:
   (a) \( f(x) = |x| \)
   (b) \( g(x) = \frac{x^2 - 1}{x + 2} \)

Solution: The critical points are the points where the derivative does not exist, or where the derivative is 0.
(a) The only critical point is \( x = 0 \).
(b) \( -2, -2 + \sqrt{3}, -2 - \sqrt{3} \)

For more practice problems, see the suggested problems on the homework handouts.