Problem 0. Read Chapter I of *The foundations of mathematics* by I. Stewart and D. Tall. Although your solutions will not be collected, make sure that you can solve the problems at the end of the chapter.

**Main Homework Problems**

1. Prove that there are infinitely many primes of the form $4n + 3$ (i.e., in the progression $3, 7, 11, 15, 19, \ldots$).

2. Prove that
   \[
   \sum_{k=0}^{\infty} \frac{1}{2k+1} = +\infty.
   \]

3. Prove that the *probability* that a randomly chosen natural number is prime is zero.

4. Prove that if $N$ is sufficiently large, then there is at least one prime number $p$ in the interval $[N, 2N]$.

5. Exhibit 2 prime numbers $p$ which are *reducible* in the set of numbers
   \[\Omega := \{a + b\sqrt{-5} : a, b \text{ integers}\} .\]

6. Exhibit 2 prime numbers $p$ which are *irreducible* in the set of numbers
   \[\Omega := \{a + b\sqrt{-5} : a, b \text{ integers}\} .\]
   Rigorously prove that these numbers are irreducible.