

# COEFFICIENTS OF HALF-INTEGRAL WEIGHT MODULAR FORMS

JAN H. BRUINIER AND KEN ONO

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Theorem 1 and Theorem 2.3 are not stated correctly. In these two results, the integer  $M$  is required to be an odd prime. For completeness, here are the correct statements.

**Theorem 1.** *Let  $f(z) = \sum_{n=1}^{\infty} a(n)q^n \in S_{\lambda+\frac{1}{2}}(N, \chi) \cap \mathbb{Z}[[q]]$  be a half-integral weight cusp form, and let  $\chi$  be a real Dirichlet character. If  $M$  is an odd prime and there is a positive integer  $n$  for which  $\gcd(a(n), M) = 1$ , then at least one of the following is true:*

(1) *If  $0 \leq r < M$ , then*

$$\#\{0 \leq n \leq X : a(n) \equiv r \pmod{M}\} \gg_{r,M} \begin{cases} \sqrt{X}/\log X & \text{if } 1 \leq r < M, \\ X & \text{if } r = 0. \end{cases}$$

(2) *There are finitely many square-free integers, say  $n_1, n_2, \dots, n_t$ , for which*

$$f(z) \equiv \sum_{i=1}^t \sum_{m=1}^{\infty} a(n_i m^2) q^{n_i m^2} \pmod{M}.$$

*Moreover if  $\gcd(M, N) = 1$ ,  $\epsilon \in \{\pm 1\}$  and  $p \nmid NM$  is a prime with  $\left(\frac{n_i}{p}\right) \in \{0, \epsilon\}$  for each  $1 \leq i \leq t$ , then  $(p-1)f(z)$  is an eigenform modulo  $M$  of the half-integral weight Hecke operator  $T(p^2, \lambda, \chi)$ . In particular, we have*

$$(p-1)f(z) \mid T(p^2, \lambda, \chi) \equiv \epsilon \chi(p) \left(\frac{(-1)^\lambda}{p}\right) (p^\lambda + p^{\lambda-1})(p-1)f(z) \pmod{M}.$$

**Theorem 2.3.** *Suppose that  $f(z)$  and  $M$  are as in Theorem 1. If there is a positive integer  $n$  for which  $\gcd(a(n), M) = 1$ , then at least one of the following is true:*

(1) *If  $0 \leq r < M$ , then there are infinitely many integers  $n$  for which  $a(n) \equiv r \pmod{M}$ .*

(2) *There are finitely many square-free integers, say  $n_1 < n_2 < \dots < n_t$ , for which*

$$f(z) \equiv \sum_{i=1}^t \sum_{m=1}^{\infty} a(n_i m^2) q^{n_i m^2} \pmod{M}.$$

MATHEMATISCHES INSTITUT, UNIVERSITÄT KÖLN, WEYERTAL 86-90, D-50931, KÖLN, GERMANY  
E-mail address: bruinier@math.uni-koeln.de

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF WISCONSIN, MADISON, WISCONSIN, 53706  
E-mail address: ono@math.wisc.edu

Typeset by  $\mathcal{A}\mathcal{M}\mathcal{S}\text{-T}\mathcal{E}\mathcal{X}$