NOTE ADDED IN THE PROOF.

Many new results regarding mock theta functions have been obtained since this paper was accepted for publication. Section 5 of the present paper describes some of Gordon's work with McIntosh on the modular transformation properties of "infinite families" of mock theta functions. Building on the important 2002 Ph.D. thesis of Zwegers [5, 6], written under the direction of Zagier, it is now known that mock theta functions are essentially the holomorphic projections of weak Maass forms of weight $1/2$ (for example, see [2, 3]). The non-holomorphic parts of these forms are certain period integrals of classical weight $3/2$ cusp forms. In several recent papers [2, 3], the last author and Bringmann have employed these results of Gordon and McIntosh [4] to provide a complete description of a natural infinite family of such Maass forms, those which are "deformations" of Ramanujan's third order mock theta function $f(q)$. The arithmetic and analytic properties of these Maass forms lead to new partition theorems. For example, their arithmetic properties can be used to show that Dyson's original rank statistic provides a combinatorial explanation for infinite families of Ramanujan-type congruences for $p(n)$ modulo arbitrary powers of any prime $\ell \geq 5$ [3]. Their analytic properties can be used to obtain precise asymptotic information for $N(r, t; n)$, the number of partitions of n with rank congruent to $r \pmod{t}$ (see [1, 2]). For example, Bringmann and the last author employ the theory of Maass forms to prove [2] the Andrews-Dragonette Conjecture for the mock theta function $f(q)$. In the spirit of this article, we stress the importance of the transformation laws of Gordon and McIntosh [4]. Their work plays a central role in clarifying how several infinite families of mock theta functions sit within the infinite dimensional spaces of Maass forms.

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