New Results on Thickness-Two Graphs

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A graph G is said to have *thickness-t* if $E(G)$ can be partitioned into t and no fewer planar graphs. For example, if G is planar, then G has thickness-one. For another example, with the help of Kuratowski's Theorem, it is easy to see that K_5 has thickness at least two. Exercise: Show that K_5 has thickness exactly two.

A longstanding open problem is the following:

What is the largest chromatic number of any thickness-two graph?

The largest chromatic number of any thickness-two graph is known to be one of 9, 10, 11, or 12. The "9" is due to exactly one published example of a 9-critical thickness-two graph due to Thom Sulanke. The "12" is due to a straightforward argument that relies on Euler's Formula for plane graphs.

We introduce a catalog of new small 9-critical thickness-two graphs, and a construction that generates infinitely many 9-critical thickness-two graphs, thus providing ballast to the "9." In addition new families of thickness-two graphs will be defined, some of which have a known asymptotically sharp upper bound on the chromatic number. Finally, an infinite family of Catlin's will be presented for which both the thickness and chromatic number are known.

This presentation is composed of earlier work with Debra Boutin (Hamilton College Mathematics) and Thom Sulanke (Indiana University Physics), and current work with Thom Sulanke.

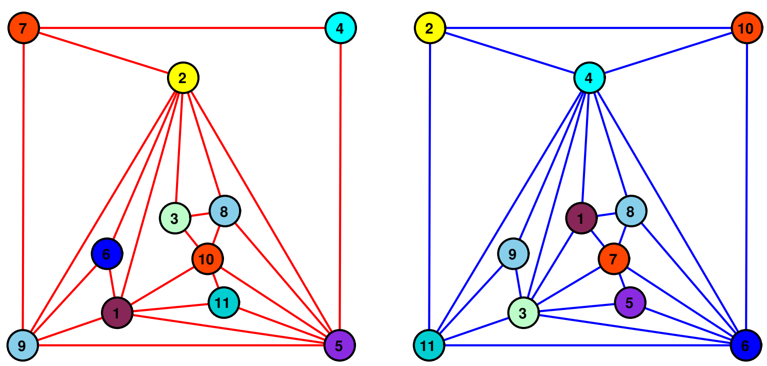

Figure 1: Thom Sulanke's thickness-two decomposition of $K_6 \vee C_5$ as a *permuted layer* graph.