Edge critical graphs with low bound of average degrees

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Abstract

A $k$-edge-coloring of a graph $G$ is a function $\phi : E(G) \mapsto \{1, \ldots, k\}$ such that any two adjacent edges receive different colors. The edge chromatic number, denoted by $\chi_e(G)$, of a graph $G$ is the smallest integer $k$ such that $G$ has a $k$-edge-coloring. Vizing’s Theorem states that the edge chromatic number of a simple graph $G$ is either $\Delta$ or $\Delta + 1$, where $\Delta$ denotes the maximum vertex degree of $G$. A graph $G$ is class one if $\chi_e(G) = \Delta$ and is class two otherwise. A class two graph $G$ is critical if $\chi_e(G - e) < \chi_e(G)$ for each edge $e$ of $G$. A critical graph $G$ is $\Delta$-critical if it has maximum degree $\Delta$. Recently D. Woodall proved that average degree of an edge-$\Delta$-critical graph is $\frac{4}{3}(\Delta + 3)$ for $8 \leq \Delta \leq 17$. We improve this result to that $\frac{5\Delta + 4}{7}$ for $8 \leq \Delta \leq 17$.

Key words: Adjacency lemma; Edge-critical graph.