

# Edge critical graphs with low bound of average degrees

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## Abstract

A  $k$ -edge-coloring of a graph  $G$  is a function  $\phi : E(G) \mapsto \{1, \dots, k\}$  such that any two adjacent edges receive different colors. The *edge chromatic number*, denoted by  $\chi_e(G)$ , of a graph  $G$  is the smallest integer  $k$  such that  $G$  has a  $k$ -edge-coloring. Vizing's Theorem states that the edge chromatic number of a simple graph  $G$  is either  $\Delta$  or  $\Delta + 1$ , where  $\Delta$  denotes the maximum vertex degree of  $G$ . A graph  $G$  is *class one* if  $\chi_e(G) = \Delta$  and is *class two* otherwise. A class two graph  $G$  is *critical* if  $\chi_e(G - e) < \chi_e(G)$  for each edge  $e$  of  $G$ . A critical graph  $G$  is  $\Delta$ -critical if it has maximum degree  $\Delta$ . Recently D. Woodall proved that average degree of an edge- $\Delta$ -critical graph is  $\frac{4}{7}(\Delta + 3)$  for  $8 \leq \Delta \leq 17$ . We improve this result to that  $\frac{5\Delta+4}{7}$  for  $8 \leq \Delta \leq 17$ .

Key words: Adjacency lemma; Edge-critical graph.